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Suppressing chaos in a simplest autonomous memristor-based circuit of fractional order by periodic impulses

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ABSTRACT

In this paper, the chaotic behavior of a simplest autonomous memristor-based circuit of fractional order is suppressed by periodic impulses applied to one or several state variables. The circuit consists of two passive linear elements, a capacitor and an inductor, as well as a nonlinear memristive element. It is shown that by applying a sequence of adequate (identical or different) periodic impulses to one or several variables, the chaotic behavior can be suppressed. Impulse values and control timing are determined numerically, based on the bifurcation diagram with impulses as bifurcation parameters. Empirically, the probability to have a reasonably wide range of impulses to suppress chaos is quite large, ensuring that chaos suppression can be implemented, as demonstrated by several examples presented.

1. Introduction

The simplest autonomous memristor-based chaotic circuit (SCC) of integer order, presented by Muthuswamy and Chua in [1], consists of only three circuit elements. As shown in Fig. 1, there are two energy-storage passive and linear elements (an inductor and a capacitor), and a nonlinear active memristor. In this way, the required circuit elements to generate chaos reduces to three, giving “the simplest possible circuit in the sense that we also have only one *locally-active* element, the memristor” [1] (see [2] for the notion of local activity).

The existence of memristor was stipulated by Chua in 1971 in his seminal paper “The missing circuit element” [3]. From a circuit-theoretic point of view, he postulated that there are four fundamental circuit variables, namely the voltage v, charge q, flux linkage φ and current i, and six two-variable combinations of those elements, as shown in Table 1 [3,4]. “From the logical as well as axiomatic points of view, it is necessary for the sake of completeness to postulate the existence of a fourth basic two-terminal circuit element which is characterized by a $\varphi - q$ curve” [3], filling the missing nonlinear relationship between charge q and flux φ, $M(\varphi, q) = 0$ (Table 1).

The term *memristor*, coined by Chua, also reflects the fact that it behaves somewhat like a nonlinear resistor with memory.

The real existence of this device was established in 2008, when a physical prototype of a two-terminal device behaving as memristor was announced in Nature [5], after Williams’s group in the HP Labs reported it on 30 April 2008. They proved the existence of a fourth basic element in integrated circuits by realizing the world-first memristor, characterizing the memristor as being “a contraction of memory resistor, because that is exactly its function: to remember its history”.

As shown by Chua, memristor can replace a circuit of over 15 transistors and several other passive elements,

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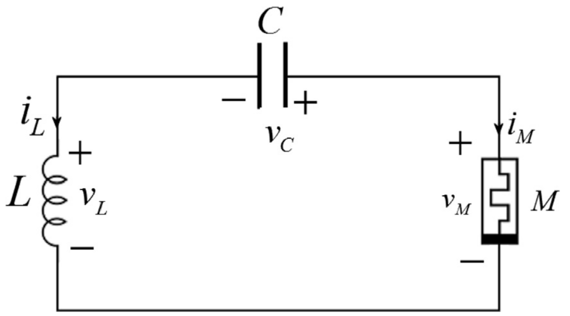

Fig. 1. Simplest autonomous memristor-based chaotic circuit, as presented in [1].

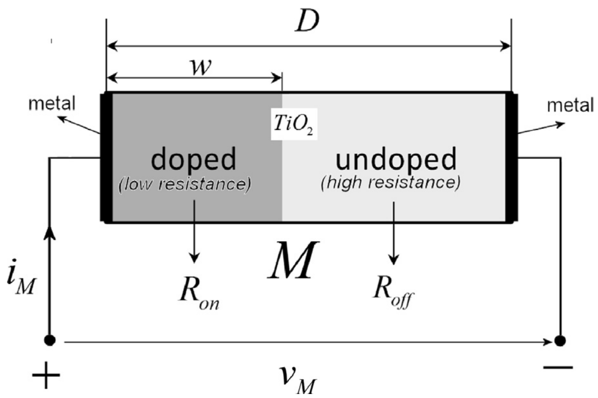

Fig. 2. Scheme of titanium-dioxide (TiO_2) memristor (adapted from [5]).

Table 1
Six possible 2-variable relationships.

Combinations of q, v, φ, i	Relationships
(v, i)	$v = Ri$
(φ, i)	$\varphi = Li$
(q, i)	$q(t) = \int_{-\infty}^{t} i(\tau)d\tau$
(q, v)	$q = Cv$
(φ, v)	$\varphi = \int_{-\infty}^{t} v(\tau)d\tau$
(φ, q)	memristor: $M(\varphi, q) = 0$

especially in small (molecular or cellular) scales. Therefore, it is useful for a large number of potential applications, generating great interest from the scientific community. For example, behaving functionally like synapses, memristors could be utilized in analog circuits mimicking the functions of the human brain (see e.g. [4]). Today, there are many research groups working on similar projects, for example, IBMs Blue Brain project, Howard Hughes Medical Institute's Janelia Farm, and Harvard Center for Brain Science. There are also many other applications in various areas, such as in electric circuits [6], logic circuits [7], concepts of computer memory [8], DRAM, flash, and disks [9], electroforming of metals and semiconductor oxides [10], memristor networks [11], bioelectricity modeling [12], next generation computers [13], cellular automata [11], linearized model of the pinched $i-v$ hysteresis [14], to mention only a few (more references can be found in [15]). The increasing interest in this element is strongly justified by the fact that more than 1800 papers published on the topic up to the middle of 2015 according to the Web of Science. It is also remarked that the concept of memristor was extended by Chua to the memcapacitor and meminductor [16], which also generate a lot of excitement to the field.

Here, consider the *current-controlled* (or *charge-controlled*) *ideal memristor* [3] (Fig. 2), as presented by the HP group, which is modeled by the following port and state equations respectively [5] (similarly, *voltage-controlled* memristor equations can be defined [17]):

$$M:\begin{cases} v_M(t) = R(x(t))i_M(t), & \text{(a)} \\ \dot{x}(t) = \pm k f(x(t))i_M(t). & \text{(b)} \end{cases} \qquad (1)$$

In this model, $R(x)$, called the *memristence* [3] as defined for HP's memristor [5], is a sum of the resistances of the doped and undoped regions (Fig. 2):

$$R(x) = xR_{on} + (1-x)R_{off}, \quad x = \frac{w}{D} \in (0,1), \qquad (2)$$

where x represents the internal state memristor variable, with w being the width of the doped region, referenced to as the total length D ($\approx 10nm$) of the (TiO_2-based) semiconductor film sandwiched between two metal contacts [5][1]; R_{on} and R_{off} ($R_{on} \ll R_{off}$) are the minimum and the maximum resistances respectively, to which the device can be configured (corresponding to $w = 0$ and $w = D$ respectively, see also [6,19]). In (1b), $f(x)$ is the so-called dopant drift window function, which models the internal state of the memristor, and k depends directly proportional to R_{on} and inversely proportional to D, while $\pm$ represents the memristor polarity [5].

Hereafter, for notational simplicity, unless necessary the time argument t will be dropped.

The nonlinear scalar function f defined in (1b), is necessary to compensate the differences between the experimental model and the theoretical model. Function f is continuous with which the solution existence and uniqueness of the underlying state equation are ensured. Several variants of f have been proposed[2], and one of the mostly used is [6]

$$f(x) = 1 - (2x-1)^{2p}, \qquad (3)$$

with p being a positive integer. The behavior of this function on some subintervals can be linear or nonlinear, depending on p (Fig. 3), an important parameter for calculating the fractional resistance of the ideal memristor.

In order to build the SCC, Muthuswamy and Chua [23] used a more general vector window function (1b), $f(x, i_M) = i_M - \alpha x - i_M x$, and a nonlinear memristance (1a), $R(x) = \beta(x^2 - 1)$, with α and β being real parameters. This kind of generalization of the ideal memristor (1a)

[1] Nowadays, there are several techniques to realize memristors by using different materials (see e.g. [18]).

[2] A linear approximation is presented in [20], with a nonlinear form in [21] (see also [15,22]).