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Solitons and nonlinear waves in the spiral magnetic structures

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ABSTRACT

This paper presents a procedure to integrate the sine-Gordon model against the background of the stripe domain structure. The nonlinear dynamics of solitons and dispersive waves in the helical (stripe domain) structure of a ferromagnet with the easy plane anisotropy in the magnetic field, which is perpendicular to the spiral axis, has been investigated in detail. It has been shown that the formation and motion of solitons are accompanied by the local translations of the stripe structure and by the oscillations of its domain walls, which manifest themselves as "precursors" and "tails" of the solitons. The large time behavior of the weak-nonlinear dispersive wave field generated by an initial localized perturbation of the structure has been investigated. The ways of observing and exciting the solitons in the spiral structure of magnets and multiferroics are discussed.

1. Introduction

This article covers the nonlinear dynamics of solitons and dispersive waves in the quasi-one-dimensional stripe domain structure of condensed matters. Such systems frequently occur. Adatom lattices at the surface of crystals with grooved potential relief can serve as numerous examples of the periodic incommensurable structures [1,2]. Many magnetic materials have no inversion centers; the quasi-one dimensional helical structure is their periodic ground state [3]. The twisting of the helicoid can be governed by varying an external magnetic field perpendicular to the magnetic spiral axis. The spiral ordering of the vector-director also characterizes cholesteric liquid crystals in external electric and magnetic fields [4]. Among advanced applied materials with a significant magnetoelectric coupling, multiferroics with the cycloidal magnetic structure cause special interest [5–7].

Theoretically, the ground state of the above systems can be described through a one-dimensional lattice of kinks (domain-wall structure). The kink lattice itself is a strongly nonlinear state of a magnetically ordered medium. Along with the substantial nonlinearity of basic equations, the inhomogeneous structure makes it difficult to analytically describe the solitons and waves in periodic structures. A constructive decision is possible but within simplified models taking the basic interactions into account correctly and, at the same time, allowing exact solutions. The most popular and universal model for describing the nonlinear dynamics of the magnetic stripe and crystal structures is the nonlinear sine-Gordon equation [3,8–12]

$$\partial_t^2 \Phi - \partial_z^2 \Phi + \sin \Phi = 0. \tag{1.1}$$

Depending on the particular problem, the field $\Phi(z, t)$ defines either the magnetization distribution, or the vector-director rotation, or the adatom displacement. Here z is the spatial coordinate, t is time.

Despite all solutions of Eq. (1.1) with a homogeneous asymptotic behavior of the field $\Phi(z, t)$ as $z \to \pm\infty$ have been investigated in detail by means of inverse scattering problem method or by its modifications [13,14], the nonlinear dynamics against the background of stripe

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domain structures is almost never studied. Being strongly nonlinear, the domain structure dramatically complicates the procedure to integrate a model. Underlying the inverse scattering problem method, the conjugate condition for analytical functions (the classical Riemann problem of the theory of functions of a complex variable) should be formulated not on the complex plane of the spectral parameter, as it was in the case of the uniform ground state of a medium, but on the Riemann surface. The latter is associated with the background structure and turns out to be topologically equivalent to a torus.

We note here, that widely used finite-gap integration is hardly effective for studying the nonlinear excitations in the periodic structure, as it leads to complicated transcendental relations and multi-dimensional theta-functions [15]. As a rule, such integration gives complex solutions of the initial model. To extract the manifold of real solutions is a difficult mathematical problem. Moreover, the finite-gap-integration is not appropriate for studying dispersive nonlinear waves, related to the continuous spectrum of the inverse scattering transform.

In [16,17] the soliton excitations on the background of spiral structure have been analyzed by Darboux and Bäcklund methods. In [18,19] the technique of “dressing” (modification of the inverse scattering problem), which allows to make complete analysis of solitons and waves in the spiral structure at localized initial conditions and boundary conditions at infinity, has been formulated. “Dressing” technique gives the real solutions of the model and allows to express the final formulae in terms of certainly studied elliptic functions. It was shown, that the formation and the motion of solitons are always accompanied by the macroscopic translations of the structure. These translations explicitly present in the boundary conditions of the problem and determine the internal structure of the solitons. Mathematically, the scheme of integration for the model (1.1) is similar to integration of the Landau–Lifshitz equation for a quasi-one-dimensional ferromagnet with the easy axis anisotropy against the background of the nonlinear precession wave of a large amplitude (see [20–23]).

In the present paper we give the complete analysis of the nonlinear dynamics of solitons and dispersive waves in the quasi-one-dimensional spiral structure of a ferromagnet without inversion center in the framework of sine-Gordon model. We discuss the possibilities of observing and exciting the solitons in the spiral structure. We investigate the behavior of dispersive wave field, generated by localized perturbation of the spiral structure at large times.

2. The sine-Gordon model for a spiral structure

Let us describe the magnetization distribution in a ferromagnet helical structure by the vector field $\mathbf{M}(z,\ t)$, where $\mathbf{M}^2 = M_0^2 = \text{const}$; z is spatial coordinate, t is time. The energy density for the quasi-one-dimensional ferromagnet without the inversion center with an easy plane anisotropy (the xOy-plane) in the constant external magnetic field $\mathbf{H} = (H, 0, 0)$ $(H > 0)$ is written as follows [17–19]:

$$\tilde{w} = \frac{\alpha}{2}(\partial_z \mathbf{M})^2 + \kappa(M_1\partial_z M_2 - M_2\partial_z M_1) + \frac{\beta}{2}M_3^2 - M_1 H. \tag{2.1}$$

Here $\alpha,\ \beta > 0$ and κ are the constants of exchange interaction, magnetic anisotropy and Dzyaloshinskii interaction, respectively. At $\mathbf{H} = 0$ the Dzyaloshinskii interaction (Lifshitz invariants) in energy (2.1) yields ideal helical ordering. The vector $\mathbf{M}(z,\ t)$ lies in the xOy plane and, upon displacement along the Oz-axis, it rotates such that a spiral structure appears, the period l_0 of which is incommensurate with the lattice parameter a and exceeds it many times, i.e. $l_0 \backsim \alpha/\kappa \gg a$. The external field $\mathbf{H}$ tends to arrange magnetic moments of atoms in the xOy plane along the Ox directions. Due to the competition of the opposite trends, extended regions of the width L_0 (domains) are formed along the Oz axis. The magnetization distribution within these regions remains nearly homogeneous. The width of the domain wall in the vicinity of the critical field $H < H_c = (\kappa\pi/4)^2 M_0/\alpha$ is $l_0 \backsim \alpha/\kappa \backsim \sqrt{\alpha M_0/H_c} \ll L_0$. The spiral magnetization turn is accomplished within the domain walls. At $H > H_c$ all the system has the commensurable ferromagnetic ordering.

For the real materials with a spiral structure the parameters of the problem satisfy inequality [17,23,24]:

$$\frac{H}{M_0} \le \frac{\kappa^2}{\alpha} \ll \beta.$$

When this restriction is valid, the Landau–Lifshitz equations for a quasi-one-dimensional ferromagnet are reduced to the sine-Gordon model [17,23]:

$$\partial_{t'}^2\Phi - \partial_{z'}^2\Phi + \sin\Phi = 0. \tag{2.2}$$

Here we use the dimensionless variables $z' = z\sqrt{H/(\alpha M_0)}$, $t' = \gamma\sqrt{\beta H M_0}\,t$. Below, strokes “′” will be omitted. In the first approximation the magnetization distribution is

$\mathbf{M} \approx M_0(\cos\Phi, \sin\Phi, 0)$.

In dimensionless variables the energy density $w = \tilde{w}/(M_0 H)$ of a quasi-one-dimensional ferromagnet takes the form:

$$w = \frac{1}{2}[(\partial_z\Phi)^2 + (\partial_t\Phi)^2] + q\partial_z\Phi + (1 - \cos\Phi), \tag{2.3}$$

where $q = \kappa\sqrt{M_0/(\alpha H)}$.

Depending on the value of q, the minimum of energy (2.3) corresponds either homogeneous distribution of the order parameter $\Phi = 0 \pmod{2\pi}$, or the periodic structure:

$$\begin{gathered}\Phi = \varphi_0(\chi) = \pi - 2\,\mathrm{am}(\chi, k), \quad \chi \equiv z/k;\\ \cos\frac{\varphi_0}{2} = \mathrm{sn}(\chi, k), \quad \sin\frac{\varphi_0}{2} = \mathrm{cn}(\chi, k),\\ \frac{\partial_\chi\varphi_0}{2} = -\mathrm{dn}(\chi, k),\end{gathered} \tag{2.4}$$

where $\mathrm{sn}(\chi,\ k)$ and etc. are the Jacobi elliptic functions with modulus k $(k^2 \le 1)$ [25–27].

An average energy on the one period of spiral structure is

$$\bar{W} = \frac{1}{L_0}\int_0^{L_0} \mathrm{d}z\, w(z) = -\frac{\pi q}{K k} - 2\left(\frac{k'}{k}\right)^2 + \frac{4E}{K k^2}.$$

Here $E = E(k)$ is complete elliptic integral of the second kind. Minimization of energy $\bar{W}$ as a function of k gives the equation:

$$\pi q k - 4E = 0. \tag{2.5}$$