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# Numerical investigation of the strength of collapse of a harmonically excited bubble

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ABSTRACT

The nonlinear dynamics of an acoustically excited spherical gas bubble in water is being investigated numerically. The applied model to describe the motion of the bubble radius is the Keller–Miksis equation, a second order ordinary differential equation, which takes into account the compressibility of the liquid. During the radial oscillations of the bubble, it may enlarge and collapse violently causing high temperature and pressure or even launch a strong pressure wave at the collapse site. These extreme conditions are exploited by many applications, for instance, in sonochemistry to generate oxidising free radicals. The recorded properties, such as the very high bubble wall velocity, and maximum bubble radius of the periodic and chaotic solutions are good indicators for the strength of the collapse. The main aim is to determine the domains of the collapse-like behaviour in the excitation pressure amplitude–frequency parameter space. Results show that at lower driving frequencies the collapse is stronger than at higher frequencies, which is in good agreement with many experimental observations (Kanthale et al., 2007, Tatake and Pandit, 2002). To find all the co-existing stable solutions, at each parameter pair the model was solved numerically with a simple initial value problem solver (4th order Runge–Kutta scheme with 5th order embedded error estimation) by applying 5 randomly chosen initial conditions. These co-existing attractors have different behaviour in the sense of the collapse strength.



## 1. Introduction

A single gas bubble can be driven to oscillate in a liquid medium when it is exposed to high amplitude, high frequency acoustic wave, or in other words exposed to ultrasound. This phenomenon is usually called acoustic cavitation in the literature. The dynamics of such a bubble is an example of a highly nonlinear oscillator [3,4]. During its motion, it may enlarge and collapse violently causing extreme conditions around itself in the liquid. With an initially large bubble radius, its wall velocity can be extremely high (even 10,000 m/s), and its radius can become many orders smaller than the initial radius. This phase of the motion is called the collapse phase, in which the pressure and the temperature at the collapse site can be as high as 1000 bar and 8000 K, respectively [5]. After the collapse, the high pressure leads to the launch of a spherically symmetric pressure wave, called shock wave [6–8].

Many applications exploit the aforementioned extreme conditions generated by the collapse of acoustically excited individual bubbles. In medicine, for instance, ultrasound has several therapeutic applications. Lithotripsy is a technique where kidney stones are broken up by ultrasound-generated shock waves [9]. The collapse-generated shock waves can also ablate or remove tissue cells in a targeted area and this procedure is called histotripsy [10]. During sonoporation encapsulated microbubbles are used

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in vivo or in vitro, targeting drug delivery with moderate intensity ultrasound [11,12], just to name a few of them.

In food industry, high power ultrasound is used for food processing [13]. For example, the generated shock wave can provide efficient mixing for two immiscible liquids [14,15]. Depending on the intensity of the ultrasound, it can decrease or increase the viscosity of a liquid due to the shear stress caused by cavitation: in the case of thixotropic fluids it decreases the viscosity [16], while in some vegetable pures ultrasound changes the fibre network which increases the viscosity [17]. The high temperature at the collapse site can provide sufficient preservation: at higher temperature the bacterial cell membrane weakens enough to became less resistant to cavitational damage [18,19].

One of the great success stories in modern chemistry to increase the chemical yield is the utilisation of ultrasound, called sonochemistry. It is widely used in wastewater cleaning to generate highly reactive free radicals for oxidation processes [20–22]. Pyrolysis is the mechanism for removing volatile pollutants which is due to the generated high pressure at the collapse site [23]. With high intensity ultrasound, the generated shock waves can reduce the molecular chain length of polymers in solution, and hence their molecular weight can be reduced [24,25]. Even without strong collapse and shock wave emission the chemical activity can be increased in various processes by the turbulent fluid flow generated by the oscillations of bubbles, called acoustic microstreaming and micromixing [26,27].

On the topic of acoustically excited bubbles, several reviews [28–31] and numerical studies [4,32–37] were written in the past few years. The accumulated computational results have revealed the very complex nature of such a system, for instance, the topology of regular periodic and chaotic windows as a function of a single control parameter, or the resonance properties with respect to the driving frequency, see also [38–41]. From the application point of view, however, it is more important to get a better picture about the strength of the bubble collapse, if it exists, than the topological structure of the solutions.

Therefore, the main aim of the present paper is to investigate a harmonically excited spherical gas bubble placed in liquid water, and to perform detailed parametric studies in the excitation amplitude–frequency plane to reveal the domains of collapse-like behaviour. The employed model is the highly non-linear Keller–Miksis equation, which takes into account the liquid compressibility to a first order approximation. This model was solved with a simple initial value problem solver (4th order Runge–Kutta scheme with 5th order embedded error estimation), and after the convergence to a stable solution, several properties were recorded, such as the maximum of the bubble radius and the maximum of the bubble wall velocity, which are good indicators for the strength of the collapse. Unlike the majority of papers, here five randomly chosen initial conditions were applied to reveal the co-existing solutions, which may have different long term behaviour in the sense of the strength of the collapse. Our results showed that the intensity of the collapse increases with decreasing driving frequency, which is in good agreement with many experimental observations [1,2].

## 2. The bubble model and solution technique

We shall see that during our computations the bubble wall velocity can reach several thousands of m/s. In order to model such large amplitude, collapse-like oscillations of a single gas bubble in water, the compressibility of the liquid has to be taken into account, since it has a great impact on the collapse phase [42]. From the family of the equations, which take into account the liquid compressibility to the first order of the Mach number, the Keller–Miksis equation is the most accurate one [43]. Hence the model used to describe the evolution of the radius of the acoustically excited spherical gas bubble in time is the Keller–Miksis equation [44], modified according to [31]:

$$\left(1-\frac{\dot{R}}{c_L}\right)R\ddot{R}+\left(1-\frac{\dot{R}}{3c_L}\right)\frac{3}{2}\dot{R}^2 = \frac{1}{\rho_L}\left(1+\frac{\dot{R}}{c_L}\right)(p_L-p_\infty(t))+\frac{R}{\rho_L c_L}\frac{d(p_L-p_\infty(t))}{dt}. \tag{1}$$

This is a second order, strongly nonlinear ordinary differential equation. The model incorporates the sound radiation from the oscillating bubble. $R=R(t)$ is the bubble radius, and the dot stands for the derivative with respect of time. $\rho_L$ and $c_L$ are the density and sound speed in the liquid domain, respectively. The pressure at the bubble wall is $p_L$, and the pressure far away from the bubble is

$$p_\infty = P_\infty + p_A \sin(\omega t), \tag{2}$$

where $P_\infty$ is the static or ambient pressure, $p_A$ is the pressure amplitude and $\omega$ is the angular frequency of the periodic excitation. The relationship between the pressures inside and outside the bubble at the bubble wall can be written as

$$p_G + p_V = p_L + \frac{2\sigma}{R} + 4\mu_L\frac{\dot{R}}{R}, \tag{3}$$

where the total pressure inside the bubble is the sum of the partial pressures of the non-condensable gas content $p_G$ and the vapour pressure $p_V$. The surface tension is $\sigma$ and the liquid kinematic viscosity is $\mu_L$. The gas inside the bubble obeys a simple polytropic relationship:

$$p_G = p_{g0}\left(\frac{R_0}{R}\right)^{3n}, \tag{4}$$

where $p_{g0}$ and $R_0$ are the gas reference pressure and gas reference radius, respectively. $n=1.4$ is the polytropic exponent.

### 2.1. The unexcited system

To understand system (1)–(4) completely, it is important to see first what happens with the unexcited system, and find the possible equilibrium solutions, that is, when the bubble radius stays at rest. Therefore, let us consider all the derivatives and $p_A$ to be zero. The equation in this case reduces to (5), where $R=R_E$ is the equilibrium radius:

$$0 = p_{g0}\left(\frac{R_0}{R_E}\right)^{3n} + p_V - P_\infty - \frac{2\sigma}{R_E} \tag{5}$$