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Effect of bounded noise on chaotic motions of stochastically perturbed slowly varying oscillator

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ABSTRACT

The effect of bounded noise on the chaotic behavior of a class of slowly varying oscillators is investigated. The stochastic Melnikov method is employed and then the criteria in both mean and mean-square sense are derived. The threshold amplitude of bounded noise given by stochastic Melnikov process is in good comparison with one determined by the numerical simulation of top Lyapunov exponents. The presence of noise scatters the chaotic domain in parameter space and the larger noise intensity results in a sparser and more irregular region. Both the simple cell mapping method and the generalized cell mapping method are applied to demonstrate the effects of noises on the attractors. Results show that the attractors are diffused and smeared by bounded noise and if the noise intensity increases, the diffusion is exacerbated.

1. Introduction

The research on chaotic phenomena in nonlinear dynamical systems has been engaging the attention and interest of researchers for many decades, since Lorenz [1] proposed a remarkable system representing a flow in three-dimensional space. However, it is particularly known that noise is inevitably present in the real world when one considers the dynamics of a system. In recent years, the effects of noise on nonlinear systems exhibiting chaotic behaviors have aroused considerable attention. Bulsara et al. [2] took into account the effects of weak additive noise on chaotic attractors of the rf SQUID. The effect of noise on the evolution of cooperation was mentioned in the studies of Wang et al. [3,4]. Frey and Simiu [5,6] developed the method of Melnikov, which is a technique providing necessary conditions for the occurrence of chaos in deterministic systems, to a class of single degree of freedom system with stochastic excitation. The generalized stochastic Melnikov method was employed to derive a mean–square criterion of a periodically forced Duffing system with additive Gaussian white noise by Lin and Yim [7], which leads to a conclusion that the presence of noise lowered the threshold and enlarged the possible chaotic domain in parameter space. The effects of external or parametric bounded noise on the chaotic behavior of the Duffing oscillator have been analyzed by Xu et al. [8] and Zhu et al. [9]. As an extension to high-dimensional case, a quasi-integrable Hamiltonian system with two degree-of-freedom was employed and the stochastic Melnikov process was derived when the harmonic and the bounded noise excitations were imposed on the system [10]. Moreover, some transient dynamics, such as mean first passage time through a potential barrier and mean growth rate coefficient as a function of the noise intensity, were studied in [11,12]. Nevertheless, the noise effect on three-dimensional systems draws paltry attention. Wiggins and Holmes [13] and Wiggins and Shaw [14] obtained necessary conditions for the occurrence of chaos in a class of slowly varying oscillators in which

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perturbations were periodic. Then Simiu [15] generalized these authors' theories for the case of quasiperiodic or stochastic perturbations. Besides acquiring necessary conditions for chaos induced by stochastic perturbations, he also estimated the lower bounds for the mean time of exit from preferred regions of phase space.

The main purpose of this paper is to investigate a specific stochastically perturbed slowly varying oscillator characterized by three-dimensional system adopting the generalized Melnikov technique and to show the effect of noise on chaotic behaviors and the attractors of the system.

An outline of this paper is as follows. In Section 2, initially we briefly introduce the system and the bounded noise. Subsequently we bring Sections 2.1 and 2.2 into a discussion of the geometry of the phase space of the unperturbed and perturbed system, describing the basic perturbation results. Standard linear analysis of fixed points is carried on in Section 3 and a derivation of the stochastic Melnikov process is demonstrated in Section 4. To verify the results of our analytical approach, simulations of the governing equations are performed in Section 5, in which various numerical methods are applied. Ultimately conclusions are given in Section 6.

2. Formulation

We consider a class of third order nonautonomous systems to model vibration of a feedback controlled buckled column subjected to additive noise. Its deterministic case was first proposed by Homes and Moon [16] and then examined by Wiggins [17]. The system is given by

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_1 - x_1^3 - I + \varepsilon(-\delta x_2 + \xi(t)) \\
\dot{I} &= \varepsilon(\gamma x_1 - \alpha I) \\
\dot{\theta} &= \Omega
\end{aligned} \tag{1}$$

where $\alpha,\ \gamma,\ \delta$ are parameters, Ω is positive and ε is small and fixed. $\xi(t)$ is a bounded noise which is a harmonic function with constant amplitude and random frequency and phases, whose mathematical expression is

$$\begin{aligned}
\xi(t) &= \mu cos(\theta + \psi) \\
\psi &= \sigma B(t) + \Gamma
\end{aligned} \tag{2}$$

where $\mu,\ \sigma$ are positive constants, $B(t)$ is a unit Wiener process, Γ is a random variable uniformly distributed in $[0, 2\pi)$. $\xi(t)$ is a stationary random process in wide sense with zero mean. Its covariance function and spectral density are respectively

$$\begin{aligned}
C_\xi(\tau) &= \frac{\mu^2}{2}\exp\left(-\frac{\sigma^2\tau}{2}\right)cos\Omega\tau \\
S_\xi(\omega) &= \frac{(\mu\sigma)^2}{2\pi}\left(\frac{1}{4(\omega-\Omega)^2+\sigma^4}+\frac{1}{4(\omega+\Omega)^2+\sigma^4}\right)
\end{aligned} \tag{3}$$

The variance of the noise is $C(0) = \mu^2/2$ which implies that the noise has finite power. It is a narrow-band process when σ is small and approaches to white noise as $\sigma \to \infty$. It can be shown that the sample functions of the noise are continuous and bounded which are required in the derivation of the Melnikov function [5].

2.1. The unperturbed system

The unperturbed system is given by

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_1 - x_1^3 - I \\
\dot{I} &= 0 \\
\dot{\theta} &= \Omega
\end{aligned} \tag{4}$$

and the (x_1, x_2) component of (4) has the form of a 1-parameter family of Hamiltonian systems with its Hamiltonian function given by

$$H(x_1, x_2; I) = \frac{x_2^2}{2} - \frac{x_1^2}{2} + \frac{x_1^4}{4} + Ix_1 \tag{5}$$

Fixed points of (x_1, x_2, I) component of (4) are given by $(x_1(I), 0, I)$, where $x_1(I)$ is a solution to the equation of

$$x_1^3 - x_1 + I = 0 \tag{6}$$

For $I \in \left(-2/3\sqrt{3}, 2/3\sqrt{3}\right)$, Eq. (6) has three solutions with the intermediate one corresponding to a hyperbolic fixed point. For $I > 2/3\sqrt{3}$ and $I < -2/3\sqrt{3}$, there exists only one solution of Eq. (6) corresponding to an elliptic fixed point and for $I = \pm 2/3\sqrt{3}$, Eq. (6) has two solutions corresponding to an elliptic fixed point and a saddle-node fixed point. Since we will only be interested in hyperbolic fixed points, we denote the interval $\left(-2/3\sqrt{3}\ ,\ 2/3\sqrt{3}\right)$ as J.

The hyperbolic fixed points of the (x_1, x_2, I) component of (4) form one-manifold denoted by

$$\gamma(I) = (\bar{x}_1(I)\ ,\ 0\ ,\ I) \tag{7}$$

where $\bar{x}_1(I)$ is the intermediate root of Eq. (6) when $I \in J$. For each fixed I, i.e. for each fixed point there exists a pair of homoclinic orbits satisfying

$$\frac{x_2^2}{2} - \frac{x_1^2}{2} + \frac{x_1^4}{4} + Ix_1 = -\frac{1}{2}\bar{x}_1^2(I) + \frac{1}{4}\bar{x}_1^4(I) + I\bar{x}_1 \tag{8}$$

Thus, the phase space of (4) is shown in Fig. 1.

Therefore in the full context of (x_1, x_2, I, θ) phase space the unperturbed system (4) has a two-dimensional normally hyperbolic invariant manifold with boundary

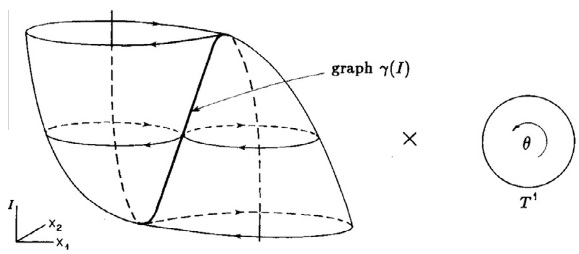

Fig. 1. Phase space of the unperturbed system (4).