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When Darwin meets Lorenz: Evolving new chaotic attractors through genetic programming

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ABSTRACT

In this paper, we propose a novel methodology for automatically finding new chaotic attractors through a computational intelligence technique known as multi-gene genetic programming (MGGP). We apply this technique to the case of the Lorenz attractor and evolve several new chaotic attractors based on the basic Lorenz template. The MGGP algorithm automatically finds new nonlinear expressions for the different state variables starting from the original Lorenz system. The Lyapunov exponents of each of the attractors are calculated numerically based on the time series of the state variables using time delay embedding techniques. The MGGP algorithm tries to search the functional space of the attractors by aiming to maximise the largest Lyapunov exponent (LLE) of the evolved attractors. To demonstrate the potential of the proposed methodology, we report over one hundred new chaotic attractor structures along with their parameters, which are evolved from just the Lorenz system alone.

1. Introduction

An important research theme in non-linear dynamics is to identify sets of differential equations along with their parameters which give rise to chaos. Starting from the advent of Lorenz attractors in three dimensional nonlinear differential equations [1,2], its several other family of attractors have been developed like Rossler, Rucklidge, Chen, Lu, Liu, Sprott, Genesio-Tesi, Shimizu-Morioka etc. [3]. Extension of the basic attractors to four or even higher dimensional systems has resulted in a similar family of hyper-chaotic systems [4]. In addition, several other structures like multi-scroll [5] and multi-wing [6] versions of the Lorenz family of attractors have also been developed by increasing the number of equilibrium points. Development of new chaotic attractors has huge application especially in data encryption, secure communication [7] etc. and in understanding the dynamics of many real world systems whose governing equations match with the template of these chaotic systems [8]. There has been several research reports on the application of master–slave chaos synchronization in secure communication [9], where the fresh set of chaotic attractors can play a big role due to their rich phase space dynamics. Here we explore the potential of automatic generation of chaotic attractors which is developed from the basic three dimensional structure of the Lorenz system.

We use the Lyapunov exponent – the most popular signature of chaos, to judge whether a computer generated arbitrary nonlinear structure of third order differential equation along with some chosen parameters exhibits chaotic motion in the phase space. Since there could be chaotic behaviour or complex limit cycles or even stable/unstable motions in the phase space, for unknown mathematical expressions of similar third order dynamical system, the computationally tractable way for the investigation of the chaos seems to be the characterisation of the observed time series. The Taken's theorem says that

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the original attractor could be reconstructed from the observation of just one state variable using the time delay embedding method [10]. In the time delay reconstruction method, the dynamics of the chaotic attractors is approximated in the phase space by plotting the observed time series and its delayed versions along orthogonal axes and finding the delay that has got maximum span in the phase space [10,11].

We predominantly report similar chaotic attractors evolved over the basic Lorenz system of equations by changing two and three state equations together using the GP. We use standard nonlinear terms in the Lorenz family like the cross product and square terms in combination with sinusoidal terms, giving rise to multiple equilibrium points and hence very complex dynamical behaviour in the phase space e.g. with infinite number of equilibrium points [12,13].

Previous attempts have looked at chaotic dynamical systems modelling through the use of genetic programming [14,15] using nonlinear autoregressive moving average with exogenous inputs or NARMAX models. The papers try to reproduce the dynamics of the Chua circuit through the nonlinear autoregressive models with different lags and orders. They also use multi-objective genetic algorithms to obtain a set of parsimonious models which can reproduce the original behaviour of the known chaotic attractor. The disadvantage of the paper is that the obtained expressions have multiple time delay terms in them, which are more complex than the ones obtained intuitively (i.e. which have three coupled first order nonlinear differential terms). Also, due to the method employed, the obtained chaotic dynamics are very similar to the original attractor and their phase portraits do not differ drastically, i.e. new types of chaotic attractors with totally different phase space dynamics are difficult to obtain using this method.

There have been attempts of evolving new single state discrete time chaotic dynamical systems using the concept of GP and study of their bifurcation diagrams [16]. Similar work has been done in [17] using evolutionary algorithms and in [18] using analytical programming techniques. The technique has also been extended to higher number of states and used for evolutionary reconstruction of continuous time chaotic systems [19]. The present paper uses a MGGP paradigm to evolve multiple expressions for the state variables simultaneously. The multi-gene approach helps in obtaining new sets of dynamical equations which can show completely different phase space dynamics. This fact is exploited in the GP algorithm to find new sets of chaotic dynamical systems by maximising the LLE.

This is the first paper of its kind that reports not only a single or a handful of attractors by varying the basic Lorenz system of equations like Rössler, Chen, Lu, Liu etc., but over hundreds of new interesting differential equation structures along with their parameters and Lyapunov exponents. The generic method proposed in this paper can be viewed as the first step towards explaining very complex chaotic motions hidden in various physical systems, which could be obtained by mixing the Lorenz equation with simple transcendental like sinusoids [20], which has got infinite number of equilibria.

2. Genetic programming to evolve new chaotic attractors

2.1. Basics of genetic programming

Genetic programming is an intelligent algorithm which is capable of automatically evolving computer programs to perform a given task [21,22]. The GP algorithm has been applied to a variety of practical applications in human competitive engineering design [23]. GP has been used for symbolic regression to fit analytical nonlinear expressions to do prediction for any given experimental input–output data set [24,25]. Essentially it can evolve the structure and parameters of a nonlinear expression by minimising the mean squared error (MSE) between the predicted and the observed values. This expression is analytic in nature and is therefore amenable to mathematical analysis. The present work exploits this paradigm to evolve analytical expressions for chaotic attractors, by trying to maximise the LLE of each of the nonlinear dynamical systems.

The GP algorithm is based on the Darwinian principle of evolution and survival of the fittest. The recently introduced multi-gene GP or MGGP [24] is used in the present study. Each state equation which the GP searches is represented by one gene. If the GP simultaneously searches for two state equations, then they are represented by two separate genes and together they constitute one individual. These genes can be represented in the form of a tree structure as shown in Fig. 1. In the beginning, the individuals are randomly initialized within the feasible space. Then they undergo reproduction, crossover and mutation to evolve fitter individuals in the subsequent generations. Crossover refers to the interchange of genetic material among the solutions. Mutation on the other hand refers to a random change within a gene itself. The crossover and mutation operations are stochastic ones and their probability of occurrence is pre-specified by the user. The tree structure representation as shown in Fig. 1 is useful for doing the cross over and mutation operations algorithmically. Generally the maximum number of levels in a tree is confined to a specific small number to decrease the bloat in the solutions and also to reduce the run time of the algorithms. Here bloat refers to the case where the GP goes on evolving very complicated nonlinear expressions, without significant increase in the performance metric (i.e. the objective function) [23].

The MGGP algorithm, as introduced in [24], uses a tree structure to represent each sub-gene and a weighted combination of these sub-genes are used to represent one individual gene. In this paper, we use the sub-genes to represent each of the state equations of the chaotic attractor, but we do not use the weights. Therefore the whole dynamical system is represented by one gene which has multiple sub-genes for each of the different state variables. So for example, if the objective is to evolve only two of the three state equations of the dynamical system, then two sub-genes would represent the overall system (since the other state equation is constant).