Chaos, Solitons & Fractals 76 (2015) 205–217

Contents lists available at ScienceDirect

Chaos, Solitons & Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

The power of unit root tests under local-to-finite variance errors

Nunzio Cappuccio [a], Diego Lubian [b,*], Mirko Mistrorigo [c]

[a] *Department of Economics and Management "Marco Fanno", via del Santo 33, 35123 Padova, Italy*
[b] *Department of Economics, University of Verona, via dell'Artigliere 19, 37129 Verona, Italy*
[c] *Poste Italiane S.p.A., Viale Europa 190, 00144 Roma, Italy*

ARTICLE INFO

Article history:
Received 10 February 2015
Accepted 15 March 2015
Available online 28 April 2015

ABSTRACT

We study the power of four popular unit root tests in the presence of a local-to-finite variance DGP. We characterize the asymptotic distribution of these tests under a sequence of local alternatives, considering both stationary and explosive ones. We supplement the theoretical analysis with a small simulation study to assess the finite sample power of the tests. Our results suggest that the finite sample power is affected by the α-stable component for low values of α and that, in the presence of this component, the *DW* test has the highest power under stationary alternatives. We also document a rather peculiar behavior of the *DW* test whose power, under the explosive alternative, suddenly falls from 1 to zero for very small changes in the autoregressive parameter suggesting a discontinuity in the power function of the *DW* test.

1. Introduction

This paper is concerned with the power of unit root tests under local departures from the maintained hypothesis of finite variance of the error term. Following the approach proposed by Amsler and Schmidt [3], and used by Cappuccio and Lubian [6], we provide expressions for the asymptotic distributions of unit root tests under a sequence of local alternatives from the unit root null hypothesis under the assumption that the error term of a driftless random walk belongs to the normal domain of attraction of a stable law in any finite sample but has finite variance in the limit as $T \uparrow \infty$.

A setup of local departures from finite variance is interesting because it allows us to investigate the behavior of unit root tests in borderline or near borderline cases between finite and infinite variance. This kind of robustness analysis may indeed be relevant in practical settings where the existence of the variance is dubious such as, for example, in the analysis of financial time series. It is well-known that the empirical distribution of financial asset returns is often characterized by fat tails suggesting the relevance of non-gaussian stable laws. However, the empirical evidence in favor of the stable model is not clear-cut [11] so that the local-to-finite variance approach can be useful for improving our understanding of the robustness of unit root and stationarity tests in these circumstances.

Amsler and Schmidt [3] first proposed this approach and derived the null distribution of the KPSS test, Callegari et al. [5] obtained the asymptotic distributions of *DF* type tests of unit root, and Cappuccio and Lubian [6] obtained the null distribution of additional stationarity and nonstationarity tests. In this paper we provide additional results both on the asymptotic distributions and the finite samples properties of unit root tests under a sequence of local alternatives and local-to-finite variance error term.

In the next section we derive the asymptotic distribution of four popular test of the unit root hypothesis under a sequence of stationary and explosive alternatives when the data generating process displays local-to-finite

* Corresponding author.
E-mail addresses: nunzio.cappuccio@unipd.it (N. Cappuccio), diego.lubian@univr.it (D. Lubian), mirko.mistrorigo@gmail.com (M. Mistrorigo).

http://dx.doi.org/10.1016/j.chaos.2015.03.012
0960-0779/

variance errors. In Section 3 we carry out a simulation experiment to study the finite sample power functions of the tests.

2. Asymptotic distributions under a sequence of local alternatives

Our modeling of the local-to-finite variance process follows the approach proposed in Amsler and Schmidt [3] whereby the process has infinite variance in finite samples but collapses to the standard finite variance case asymptotically. The Data Generating Process for the error term u_t is then given by

$$u_t = v_{1t} + \frac{\gamma}{aT^{1/\alpha - 1/2}} v_{2t}. \tag{1}$$

where v_{1t} is an i.i.d. process with zero mean and finite variance σ^2 and v_{2t} is also an i.i.d. process, symmetrically distributed with distribution belonging to the normal domain of attraction of a stable law with characteristic exponent α, with $\alpha \in (0;\, 2)$, denoted as $v_{2t} \in \mathcal{ND}(\alpha)$, and a can be set equal to 1 as in Amsler and Schmidt [3]. It follows that u_t exhibits infinite variance in any finite sample size but finite variance in the limit as T approaches infinity. The role played by the stable component decreases as the sample size increases even though this occurs at a slower rate as α increases. Thus, for a given γ, when α is close to 2 we need a large sample size to offset the stable component whereas for $\alpha < 1$ a relatively small sample size is required. By Donsker's theorem, it is well known that $T^{-1/2}\sum_{t=1}^{[Tr]} v_{1t} \Rightarrow \sigma W(r)$, where $\Rightarrow$ stands for the weak convergence of probability measures, and $W(r)$ is the standard Wiener process. Further, (see, for instance, [17,15]), for the partial sum process $a_T^{-1}\sum_{t=1}^{[Tr]} v_{2t}$ we have the following convergence results

$$\left(\frac{1}{a_T}\sum_{t=1}^{[Tr]} v_{2t}, \frac{1}{a_T^2}\sum_{t=1}^{[Tr]} v_{2t}^2\right) \Rightarrow (U_\alpha(r), V(r)), \tag{2}$$

where $a_T = aT^{1/\alpha}, U_\alpha(r)$ is a Lévy α-stable process on the space $D[0,1], V(r)$ is its quadratic variation process $V(r) = [U_\alpha, U_\alpha]_r = U_\alpha^2(r) - 2\int_0^r U_\alpha^- dU_\alpha$ (see [16], pg. 58, [15], eq. (11)) and $U_\alpha^-(r)$ stands for the left limit of the process $U_\alpha(\cdot)$ in r. The process $V(r)$ is a Lévy $\alpha/2$-stable process appears frequently in the asymptotic distribution of unit root tests. For $\alpha \in (0,1)$, it is non a degenerate random variable, while for $\alpha = 2$ we have $V(1) = 1$.

The main convergence result used in the paper is the following

$$\frac{1}{\sqrt{T}}\sum_{t=1}^{[Tr]} u_t \Rightarrow \sigma_1 W(r) + \gamma U_\alpha(r) \equiv Z_{\alpha,\gamma}(r)$$

whose proof follows directly from the above joint convergence and the continuous mapping theorem. A number of useful results on the limiting behavior of sample moments and partial sums of the local-to-finite variance error term have been provided by Cappuccio and Lubian [6], Lemma 2.1.

We consider the baseline case of a driftless random walk and assume that $\{y_t\}$ is generated as

$$y_t = \rho y_{t-1} + u_t, \qquad t = 1, \ldots, T \tag{3}$$

where $\rho = 1$ and that the initial condition y_0 is any random variable whose distribution does not depend on T.

We consider four test statistics for testing the null hypothesis $H_{DS} : \rho = 1$ in (3) such as $T(\hat\rho - 1)$ and the t-ratio statistics, where $\hat\rho$ is the OLS estimator of ρ in 3, proposed by Dickey and Fuller [7], the Lagrange Multiplier test (hereafter LM) proposed by Ahn [1], and the Durbin–Watson (DW) test. Formally, the t-ratio statistics, the LM and DW tests are given by

$$t_{\hat\rho} = \left(\sum_{t=2}^{T} y_{t-1}^2\right)^{1/2} (\hat\rho - 1)/s \tag{4}$$

$$LM = \frac{\left(\sum_{t=2}^{T}(y_t - y_{t-1})y_{t-1}\right)^2}{s^2\sum_{t=2}^{T} y_{t-1}^2} \tag{5}$$

$$DW = \frac{\sum_{t=2}^{T}(y_t - y_{t-1})^2}{\sum_{t=2}^{T} y_{t-1}^2} \tag{6}$$

where $s^2 = \sum_{t=2}^{T}(y_t - y_{t-1})^2/T$. As remarked by Paulauskas et al. [13], the weak convergence to stochastic integrals for sample moments of i.i.d. random vectors in the domain of attraction of a multivariate stable law with an index $0 < \alpha < 2$ has been proved by Paulauskas and Rachev [12]. The limiting behavior of the above test statistics under DGP (1) and the null hypothesis $H_0 : \rho = 1$ has been provided by Cappuccio and Lubian [6] and is reported here for completeness

$$T(\hat\rho - 1) \Rightarrow \frac{\int_0^1 Z_{\alpha,\gamma} dZ_{\alpha,\gamma}}{\int_0^1 Z_{\alpha,\gamma}^2}, \qquad t_{\hat\rho} \Rightarrow \frac{\int_0^1 Z_{\alpha,\gamma} dZ_{\alpha,\gamma}}{\left(K_\gamma(1)\gamma^2 V(1)\int_0^1 Z_{\alpha,\gamma}^2\right)^{1/2}}$$

$$LM \Rightarrow \frac{\left(\int_0^1 Z_{\alpha,\gamma} dZ_{\alpha,\gamma}\right)^2}{K_\gamma(1)\int_0^1 Z_{\alpha,\gamma}^2}, \qquad TDW \Rightarrow \frac{K_\gamma(1)}{\int_0^1 Z_{\alpha,\gamma}^2}$$

where $K_\gamma(1) = \sigma^2 + \gamma^2 V(1)$. The null distribution of the four test statistics for DGP (3) in the infinite variance case has been established by Ahn et al. [2] and for DGPs with a constant or a constant and a drift by Callegari et al. [5]. Even though the process u_t has finite variance, the limiting distributions of the unit root test statistics turns out to be a function of both the Wiener process $W(r)$ and the Lévy α-stable process $U_\alpha(r)$, so that they depend on the maximal moment exponent α and the nuisance parameters σ^2 and γ. As expected, the weight of $U_\alpha(r)$ in the asymptotic distribution increasesas with γ.

To study the power of these tests, we consider local departures from the null hypothesis assuming that the data generating process is given by

$$y_t = \rho_T y_{t-1} + u_t \tag{7}$$

where $\rho_T = e^{c/T}$ with $c < 0$, a noncentrality parameter, to focus on the stationary alternatives. When $c = 0$ we are under the null hypothesis, while for $c > 0$ the process is