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Complex dynamics and chaos control of duopoly Bertrand model in Chinese air-conditioning market

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ABSTRACT

A dynamic duopoly Bertrand model with quadratic cost function which is closer to reality and different from previous researches is discussed. The model is applied into air-conditioning market where the boundary equilibrium point is locally stable. Numerical simulations illustrate that the stability of Nash equilibrium strongly depends on the speed of adjustment of bounded rational player. The adjustment speeds and the degree of substitutability may undermine the stability of the equilibrium and cause a market structure to behave chaotically. The Lyapunov dimension of the chaos attractor is 1.9585 under some conditions. The stabilization of the chaotic behavior can be obtained by reducing the degree of substitutability. The results have an important theoretical and practical significance to Chinese air-conditioning market.

1. Introduction

After implementing the policies of providing subsidies for rural residents to purchase home appliances and subsidies for trading-in old home appliances for new ones. Domestic air conditioning market already has been into the adjustment. By the end of 2014 domestic air conditioning industry stock set a record of nearly 21 million units. De-stocking is the most pressing problem as the whole air conditioning industry. Due to the serious homogeneity and a single means of competition, price war is inevitable. Before the National Day in 2014, Gree and Midea shouted out loud slogans. Their advertising slogans are “the first air conditioning price war in 20 years” and “only tornado in 30 years”, respectively. The two cut the price of air conditioner by 30% respectively. It is generally believed that the price war launched by industry leading Gree will continue until 2015.

The complexity of Cournot models with bounded rationality, such as cyclic, bifurcation and chaos have been studied by many researchers [1–8]. Compared with the Cournot model, Bertrand model, sufficiently considering the difference among products, is popular in practice. However, few studies on the complexity of Bertrand model have been reported. References [9–13] investigated the chaotic dynamics of Bertrand model based on the linear cost functions. Peng et al. [14] analyzed a dynamic of triopoly Bertrand repeated model with the zero marginal cost.

To the authors’ knowledge, the complexity of Bertrand model with bounded rationality based on nonlinear cost functions has been rarely studied. This paper proposes a Bertrand model based on quadratic cost function which is closer to reality. We assume that the players produce similar products and adjust price based on their marginal profit of last period try to maximize their profit. Numerical simulations are provided to indicate the complexity of the system’s evolvement and the effect of the degree of substitutability on the stability of the system. Different from Fanti et al. [9], we develop a general

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duopoly Bertrand model and analyze the effect of the degree of substitutability on the prices of two oligarchs.

2. Duopoly game model

2.1. The model

The Chinese air-conditioning brands have been phased out by 95% over ten years. The share of the first two brands, namely Gree and Midea, continue to improve. The market is almost occupied by duopoly.

We assume that Midea and Gree, denoted by index 1 and 2, produce similar products in the air-conditioning market and the firms choose the optimal prices for local profit maximization. Cost function is as follows [5]: $c_i(q_i) = c_{i0} + r_i q_i + c_i q_i^2$, where $c_{i0},\ r_i,\ c_i > 0,\ i = 1,\ 2$. The price of firm i's product is $p_i(t)$ and the demand is $q_i(t)$ during period $t = 0,1,2,\ldots$ and $q_i(t)$ is determined by $p_i(t)$. The demand function is:

$$q_i(p_i, p_j) = a_i - d_i p_i + b_i p_j \quad i,\ j = 1,\ 2, \quad i \neq j \tag{1}$$

$a_i,\ d_i > 0,\ 0 \leqslant b_i \leqslant 1,\ i = 1,\ 2$. The parameter b_i signifies substitutability. "More precisely the degree of substitutability increases, or equivalently, the extent of product differentiation decreases as the parameter rises." [9, p. 74]. Thus we obtain the profit of the ith firm in the single period as:

$$\pi_i(t) = p_i(t) q_i(t) - c_{i0} - r_i q_i(t) - c_i q_i^2(t) \quad i,\ j = 1,\ 2, \quad i \neq j \tag{2}$$

In practice, the players may use more complicated expectations such as bounded rationality. Decision-making is an adjustment process on the basis of the last period game results. They determine the production price according to their marginal profit. The marginal profit the ith firm in period t is:

$$\frac{\partial \pi_i(t)}{\partial p_i(t)} = a_i(1 + 2c_i d_i) + r_i d_i - 2d_i(1 + c_i d_i)p_i(t) + b_i(1 + 2c_i d_i)p_j(t), \quad i,\ j = 1,\ 2, \quad i \neq j \tag{3}$$

Hence, the dynamic adjustment mechanism can be modeled as:

$$p_i(t+1) = p_i(t) + \alpha_i p_i(t) \frac{\partial \pi_i(t)}{\partial p_i(t)}, \quad i = 1,\ 2 \tag{4}$$

where α_i is the price adjustment speed of firm i. Substituting (3) into (4), we obtain

$$\begin{aligned} p_1(t+1) &= p_1(t) + \alpha_1 p_1(t)[A_1 - B_1 p_1(t) + D_1 p_2(t)] \\ p_2(t+1) &= p_2(t) + \alpha_2 p_2(t)[A_2 - B_2 p_2(t) + D_2 p_1(t)] \end{aligned} \tag{5}$$

where $A_i = a_i(1+2c_i d_i) + r_i d_i$, $B_i = 2d_i(1+c_i d_i)$, $D_i = b_i(1+2c_i d_i)$, $i = 1,\ 2$.

2.2. Equilibrium points and local stability

In order to study the qualitative behavior of solutions of the nonlinear difference Eq. (5), we define the equilibrium points of the dynamic duopoly game as the nonnegative fixed points of system (5), i.e. the solution of $p_i(t+1) = p_i(t)$, (i = 1,2). Thus the four fixed points, such as $E_0(0,0)$, $E_1(0, A_2/B_2)$, $E_2(A_1/B_1, 0)$, $E^*(p_1^*, p_2^*)$, are gained, where $p_i^* = (A_i B_j + A_j D_i)/(B_i B_j - D_i D_j)$, $i,\ j = 1,\ 2,\ i \neq j$. It is obviously that E^*, different from the other boundary equilibriums, is the unique Nash equilibrium point.

To consider the local stability of the equilibrium points, we estimate the Jacobian matrix J of (5) as follows:

$$J = \begin{pmatrix} 1 + \alpha_1[A_1 - 2B_1 p_1(t) + D_1 p_2(t)] & \alpha_1 D_1 p_1(t) \\ \alpha_2 D_2 p_2(t) & 1 + \alpha_2[A_2 - B_2 p_2(t) + D_2 p_1(t)] \end{pmatrix}.$$

The local stability analysis of the four fixed points of (5) can be carried out by considering the Jacobian matrix J. E_0 is an unstable node while E_1 and E_2 are both saddle points [9].

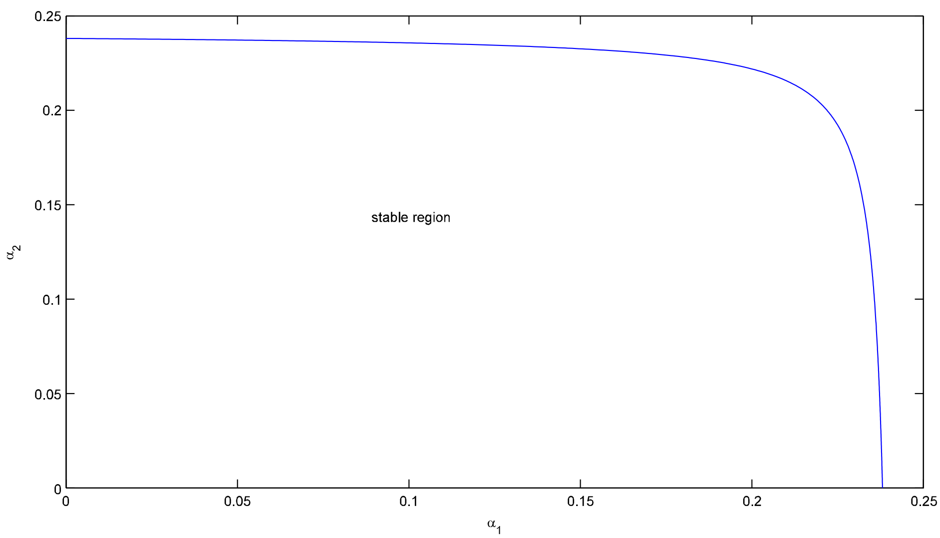

Fig. 1. The stable region of Nash equilibrium point in the plane (α_1, α_2).