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Synchronization in nonlinear oscillators with conjugate coupling

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ABSTRACT

In this work, we investigate the synchronization in oscillators with conjugate coupling in which oscillators interact via dissimilar variables. The synchronous dynamics and its stability are investigated theoretically and numerically. We find that the synchronous dynamics and its stability are dependent on both coupling scheme and the coupling constant. We also find that the synchronization may be independent of the number of oscillators. Numerical demonstrations with Lorenz oscillators are provided.

1. Introduction

The study of synchronization in coupled periodic oscillators has been active since the early days of physics [1,2]. Chaos implies sensitive dependence on initial conditions, with nearby trajectories diverging exponentially, and the synchronization among chaotic oscillators has become a topic of great interest since 1990 [3,4]. The general theories on complete synchronization in which the distance between states of interacting identical chaotic units approaches zero for $t \to \infty$ have been well framed [5–7]. In these theories, chaotic oscillators interact with each other through the same (nonconjugate) variables of different oscillators. However, coupling via dissimilar (conjugate) variables is also natural in real situations [8,9]. One example is the coupled-semiconductor-laser experiments by Kim and Roy [10], where the photon intensity fluctuation from one laser is used to modulate the injection current of the other, and vice versa. In the nonconjugate coupling case, the interaction vanishes with the buildup of complete synchronization and the synchronous state is a solution of isolated system. In contrast, the interaction in the conjugate coupling case may stay nonzero even when oscillators are synchronized.

The conjugate coupling has been used to realize the amplitude death [11,12] in coupled identical units, the phenomenon in which unstable equilibrium in isolated unit becomes stable with the assistance of coupling, in several recent works [13–15]. Interestingly, the realized amplitude death in those works is indeed one special type of synchronization. Then questions arise: Can synchronization in chaotic oscillators with conjugate coupling be realized? What is the synchronous state in chaotic oscillators with conjugate coupling and how about its stability?

The main goal in this work is to theoretically investigate the synchronous dynamics and its stability in a ring of identical chaotic oscillators with conjugate coupling by following the methods in Refs. [5,7]. The statements are demonstrated through numerical simulations with the Lorenz oscillators. We also show that the statements are valid for regular random networks in which each oscillator has the same number of neighbors.

2. Analysis

The model we consider takes the general form

$$\dot{\mathbf{x}}_i = \mathbf{f}(\mathbf{x}_i) + \epsilon(\mathcal{D}_2\mathbf{x}_{i+1} - \mathcal{D}_1\mathbf{x}_i) + \epsilon(\mathcal{D}_2\mathbf{x}_{i-1} - \mathcal{D}_1\mathbf{x}_i), \qquad (1)$$

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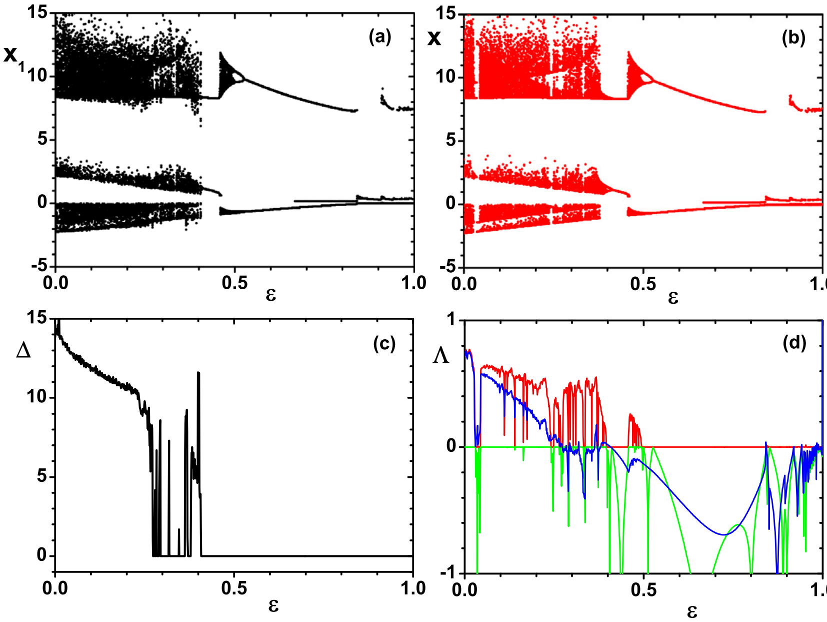

Fig. 1. (a) The bifurcation diagram of one oscillator in a pair of coupled Lorenz oscillators. (b) The bifurcation diagram of the synchronous motion which follows Eq. (2) but with 2ϵ replaced by ϵ. (c) The synchronization error Δ is plotted against the coupling constant, which shows that the synchronization error depends on ϵ in a non-monotonic way. (d) The first two largest Lyapunov exponents of the synchronous motion ($\Lambda_1^{(2)}$ in red and $\Lambda_2^{(2)}$ in green) and the largest Lyapunov exponent $\Lambda_1^{(1)}$ of the transversal mode (in blue) are plotted against ϵ. $\sigma = 10$, $r = 28$, and $\beta = 1$. The matrices D_1 and D_2 are presented in the text. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

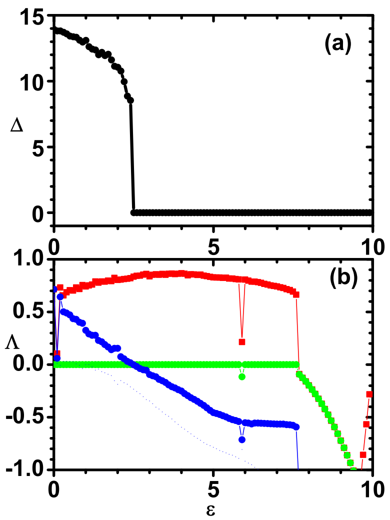

Fig. 2. The dynamics of a pair of Lorenz oscillators with D_1 and D_2 presented in the text. (a) The synchronization error Δ is plotted against the coupling constant. (b) The first two largest Lyapunov exponents of the synchronous motion ($\Lambda_1^{(2)}$ in red and $\Lambda_2^{(2)}$ in green) and the largest Lyapunov exponent $\Lambda_1^{(1)}$ of the transversal mode (in blue) are plotted against the coupling constant. $\sigma = 10$, $r = 28$, and $\beta = 1$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

where $\mathbf{x}_i \in R^n$ $(i = 1, 2, \ldots, N)$, $\mathbf{f} : R^n \rightarrow R^n$ is nonlinear and capable of exhibiting rich dynamics such as chaos. The periodic boundary conditions are imposed on Eq. (1). The parameter ϵ is a scalar coupling constant. $\mathcal{D}_1$ and $\mathcal{D}_2$ are constant matrices describing coupling schemes. When $\mathcal{D}_1 = \mathcal{D}_2$, the interaction terms become $\mathcal{D}_1(\mathbf{x}_{i+1} + \mathbf{x}_{i-1} - 2\mathbf{x}_i)$ and the ordinary non-conjugate coupled oscillators are recovered in which oscillators interact with each other through the same variables.

Now we are interested in the synchronous state; the state resides on a synchronous manifold defined by $M = \{(\mathbf{x}_1, \ldots, \mathbf{x}_N) : \mathbf{x}_i = \mathbf{s}(t)\}$ where $\mathbf{s}(t)$ obeys the equation of motion

$$\dot{\mathbf{s}} = \mathbf{f}(\mathbf{s}) + 2\epsilon(\mathcal{D}_2 - \mathcal{D}_1)\mathbf{s}. \tag{2}$$

To be noted that the synchronous state is not the solution of the isolated oscillator any more and its dynamics depends on both the coupling constant ϵ and the matrices $\mathcal{D}_1$ and $\mathcal{D}_2$. Introducing perturbation $\xi = \xi_1, \ldots, \xi_N$ to the synchronous state, we linearize equation (1) and have

$$\frac{d}{dt}\xi = I \otimes (D\mathbf{f}(\mathbf{s}) - 2\epsilon\mathcal{D}_1)\xi + \epsilon\mathbf{C} \otimes \mathcal{D}_2\xi. \tag{3}$$

$D\mathbf{f}(\mathbf{s})$ is the Jacobian matrix of $\mathbf{f}$ at $\mathbf{s}$ and I is the $N \times N$ unit matrix. The coupling matrix $\mathbf{C}$ is an $N \times N$ matrix with zero elements except that $c_{i,i+1} = c_{i-1,i} = 1$, which describes the interaction among oscillators. The eigenvalues and eigenvectors of C satisfy $C\phi_i = \lambda_i\phi_i$. By expanding ξ over the eigenvectors of C, we have $\xi = \Sigma_{i=1}^{N}\eta_i\phi_i$ where η_i are time-dependent coefficients. Substituting the expansion into Eq. (3) and equating the coefficient for each ϕ_i, we have

$$\dot{\boldsymbol{\eta}}_i = [D\mathbf{f}(\mathbf{s}) - 2\epsilon\mathcal{D}_1 + \epsilon\lambda_i\mathcal{D}_2]\eta_i,$$
$$i = 1, 2, \ldots, N, \tag{4}$$