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Collective behavior of chaotic oscillators with environmental coupling

C. Quintero-Quiroz [a], M.G. Cosenza [b,*]

[a] Departament de Física i Enginyeria Nuclear, Universitat Politécnica de Catalunya, 08222 Terrassa, Barcelona, Spain
[b] Centro de Física Fundamental, Universidad de Los Andes, Mérida, Mérida 5251, Venezuela

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ABSTRACT

We investigate the collective behavior of a system of chaotic Rössler oscillators indirectly coupled through a common environment that possesses its own dynamics and which in turn is modulated by the interaction with the oscillators. By varying the parameter representing the coupling strength between the oscillators and the environment, we find two collective states previously not reported in systems with environmental coupling: (i) nontrivial collective behavior, characterized by a periodic evolution of macroscopic variables coexisting with the local chaotic dynamics; and (ii) dynamical clustering, consisting of the formation of differentiated subsets of synchronized elements within the system. These states are relevant for many physical and biological systems where interactions with a dynamical environment are frequent.

Many physical, biological, and social systems exhibit global interactions; i.e., all the elements in the system are subject to a common influence. These systems have been widely studied in many theoretical and experimental models [1–14]. A global interaction may consist of an external field acting on the elements, as in a driven (or unidirectionally coupled) dynamical system; or it may originate from the mutual interactions between the elements, in which case, we refer to an autonomous dynamical system. Recently, there has been interest in the investigation of systems of dynamical elements subject to a global interaction through a common environment or medium that possesses its own dynamics. In this case, the state of each element in the system influences the environment, and the state of the environment in turn affects the elements. This type of global interaction has been denominated as environmental coupling [15–18]. Examples of such systems include chemical and genetic oscillators where coupling is through exchange of chemicals with the surrounding medium [19–21], ensembles of cold atoms interacting with a coherent electromagnetic field [22], coupled circadian oscillators due to common global neurotransmitter oscillation [23], a crowd walking on the Millennium Bridge [24], and quorum-sensing in delay-coupled lasers [25]. Since the elements are not directly interacting with each other but through a common medium, this configuration has also been called indirect coupling [26,27] or relay coupling [28].

In this paper we investigate the collective behavior arising in a system consisting of many chaotic oscillators subject to environmental coupling. The presence of local chaos and the large number of oscillators allow the emergence of collective states not observed in previous models with periodic oscillators or with just two oscillators. By varying the coupling strength between the chaotic oscillators and the environment, we find these collective states: (i) nontrivial collective behavior, i.e., non-statistical fluctuations in the mean-field of the ensemble, manifested by a periodic evolution of macroscopic variables coexisting with the local chaotic dynamics [29,30]; and (ii) dynamical clustering, i.e., the formation of differentiated subsets of synchronized elements within the system [31].

* Corresponding author.

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We consider a system of N chaotic Rössler oscillators coupled through a common environment that can receive feedback from the system,

$$\dot{x}_i = -y_i - z_i + \varepsilon_2 w, \\ \dot{y}_i = x_i + a y_i, \\ \dot{z}_i = b + z_i(x_i - c), \tag{1}$$

$$\dot{w} = -\lambda w + \frac{\varepsilon_1}{N}\sum_{j=1}^{N} x_j, \tag{2}$$

where $x_i(t), y_i(t), z_i(t)$ describe the state variables of oscillator $i = 1, 2, \ldots, N$, at time t; $w(t)$ represents the state of the environment at t; a, b, c are parameters of the local dynamics; the parameter ε_2 measures the strength of the global influence from the environment to the oscillators; and ε_1 represents the intensity of the global feedback to the environment. The damping parameter λ characterizes the intrinsic dynamics of the environment, which decays in time in absence of feedback from the oscillators. The form of the global coupling in the system Eqs. (1) and (2) is non-diffusive.

A complete synchronized state in the system at time t corresponds to $x_i(t) = x_j(t), y_i(t) = y_j(t), z_i(t) = z_j(t), \ \forall i, j$. The occurrence of stable synchronization in the system Eq. (1) can be numerically characterized by the asymptotic time-average $\langle\sigma\rangle$ of the instantaneous standard deviations of the distribution of state variables, defined as

$$\langle\sigma\rangle = \frac{1}{T-\tau}\sum_{t=\tau}^{T}\sigma(t), \tag{3}$$

$$\sigma(t) = \left[\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{X})^2 + (y_i - \bar{Y})^2 + (z_i - \bar{Z})^2\right]^{1/2}, \tag{4}$$

where τ is a discarded transient time, and the mean values are defined as

$$\bar{X}(t) = \frac{1}{N}\sum_{j=1}^{N} x_j(t), \tag{5}$$

$$\bar{Y}(t) = \frac{1}{N}\sum_{j=1}^{N} y_j(t), \tag{6}$$

$$\bar{Z}(t) = \frac{1}{N}\sum_{j=1}^{N} z_j(t). \tag{7}$$

Then, a complete synchronization state corresponds to a value $\langle\sigma\rangle = 0$. In practice, we use the numerical criterion $\langle\sigma\rangle < 10^{-7}$ as a synchronization condition.

On the other hand, the instantaneous phase of the trajectory of oscillator i projected on the plane (x_i, y_i) can be defined as

$$\phi_i(t) = \tan^{-1}\left(\frac{y_i(t)}{x_i(t)}\right). \tag{8}$$

To characterize a collective state of phase synchronization on the plane (x, y), we calculate the asymptotic time-average quantity

$$\Phi = \frac{1}{T-\tau}\sum_{t=\tau}^{T}\left[\left(\frac{1}{N}\sum_{j=1}^{N}\sin\phi_i(t)\right)^2 + \left(\frac{1}{N}\sum_{j=1}^{N}\cos\phi_i(t)\right)^2\right]. \tag{9}$$

Then, a collective phase synchronization state corresponds to a value $\Phi = 1$.

We have fixed the local parameters in Eqs. (1) and (2) at values the $a = b = 0.1$ and $c = 18$, for which a Rössler oscillator displays chaotic behavior. Then, we numerically integrate the system Eqs. (1) and (2) with given size N and a given value of the damping parameter λ, for different values of the coupling parameters ε_1 and ε_2. We employ a fourth-order Runge–Kutta scheme with fixed integration step $h = 0.01$. The initial conditions for the variables x_i, y_i were randomly distributed with uniform probability on the interval $[-20, 20]$, and those for the variables z_i on the interval $[0, 5], \quad \forall i$.

Fig. 1(a) shows the statistical quantities $\langle\sigma\rangle$ and Φ as functions of ε_1 for the system Eqs. (1) and (2), with

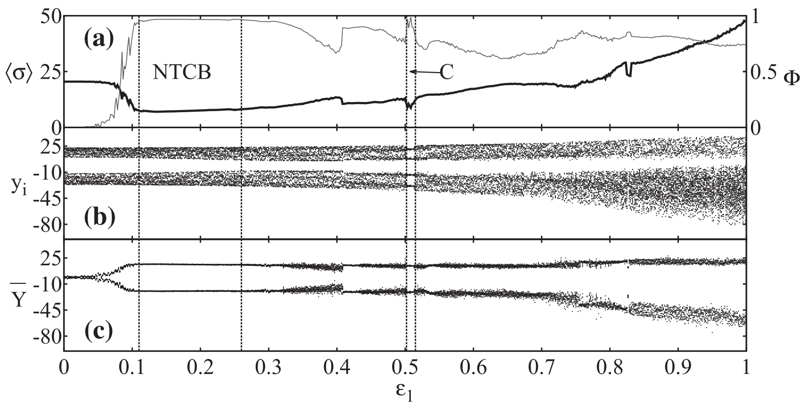

Fig. 1. (a) The quantities $\langle\sigma\rangle$ (thick line, left vertical axis) and Φ (thin line, right vertical axis) as functions of the coupling strength ε_1, for the system Eqs. (1) and (2) with $\varepsilon_1 = \varepsilon_2$. The labels NTCB and C indicate the regions of the coupling where nontrivial collective behavior and dynamical clustering occur, respectively. Fixed parameters values are: $a = b = 0.1, c = 18, \lambda = 1, N = 10^3, \tau = 10^3, T = 7 \times 10^3$. (b) Bifurcation diagram of the values y_i, when $x_i = 0$ on the plane (x_i, y_i), for one oscillator as a function of ε_1. (c) Bifurcation diagram of the component $\bar{Y}$ of the mean field, when $\bar{X} = 0$ on the plane $(\bar{X}, \bar{Y})$, as a function of ε_1. For each value of ε_1, 300 consecutive values of y_i and $\bar{Y}$ have been plotted in (b) and (c), after discarding the transient time τ.