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Distributional chaos occurring on measure center

Lidong Wang [a,b], Yan Li [a,c,*], Jianhua Liang [a]

[a] Mathematics School and Institute, Jilin University, Changchun 130012, China
[b] School of Science, Dalian Nationalities University, Dalian 116600, China
[c] School of Mathematics, Physics and Biological Engineering, Inner Mongolia University of Science and Technology, Baotou 014010, China

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ABSTRACT

In studying a dynamical system, we know that there frequently exist some kinds of disturbance or false phenomenon, namely that some chaotic sets are included in the Borel sets with absolute measure zero. From the viewpoint of ergodic theory, a Borel set with absolute measure zero is negligible. In order to remove these disturbance and false phenomenon, reference Zhou (1993) introduced a concept of measure center. This paper is concerned with distributional chaos occurring on measure center. We draw three conclusions: (i) the one-sided shift has an uncountable distributionally scrambled set which is included in the set of all weakly almost periodic points but not in the set of almost periodic points; (ii) we give a sufficient condition for the compact dynamic system (X, f) to exhibit distributional chaos on measure center via semiconjugacy; (iii) the one-sided shift on the measure center is distributionally chaotic in sequence.

1. Introduction

Since Li and Yorke first gave the definition of chaos by using strict mathematical language in 1975 [2], the study of chaos plays an important role in modern science nowadays. The influence has covered all scientific disciplines, not only natural sciences, but also several social science, such as economics, management science, and philosophy. The theory of chaos shows that a simple definite system can produce complicated features. As scientists' task is to clarify the essence of complexity, the chaotic systems with irregularly complex dynamical behaviors naturally become one common subject. However, depending on different perspective and understanding, the various concepts of chaos have been given, such as Devaney chaos [3], P-chaos [4], Kato's chaos [5], Wiggins chaos [6], Matelli chaos [7], distributional chaos [8], and ω-chaos [9], etc. Among them, distributional chaos has some actual significance. And more and more researchers give their attention to the properties of distributional chaos. Each definition tries to describe some kind of unpredictability of the system. Therefore, there exists much ambiguity in academic intercourse of different fields. At the same time, this situation cannot be tolerated in mathematical field which is based on strict mathematical definitions. Therefore, it is very significant to further explore the essence of chaos, unify the definition of chaos, and discuss the inner relations between the different definitions of chaos. Hence, in order to establish a satisfactory definitional and terminological framework for chaotic system, we need to reveal the inner link between the various concepts which characterize the complexity. And topological entropy and ergodic measure are the most important ones in these concepts. From the viewpoint of measure theory, a Borel set with absolute measure zero is negligible, and any phenomenon occurring on it is unimportant or false. In order to obtain a subsystem which not only removes all the disturbances but also preserves the very important dynamical behavior, Zhou [1] introduced the concept of measure center, and pointed out that

* Corresponding author at: Mathematics School and Institute, Jilin University, No. 2699, Qianjin Street, Changchun 130012, China. Tel.: +86 041187656232.

E-mail address: lyll1234@163.com (Y. Li).

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all of important dynamical behaviors of a compact system occur on its measure center. And he also showed the following level of Li–Yorke chaos.

(i) $f : X \to X$ is chaotic but $f|_{\Omega(f)} : \Omega(f) \to \Omega(f)$ is not;
(ii) $f|_{\Omega(f)} : \Omega(f) \to \Omega(f)$ is chaotic but $f|_{M(f)} : M(f) \to M(f)$ is not;
(iii) $f|_{M(f)} : M(f) \to M(f)$ is chaotic;

where $\Omega(f)$ denotes non-wandering set of f, and $M(f)$ denotes the measure center of f. He also proved that $M(f) = \overline{W(f)}$, where $W(f)$ denotes the weakly almost periodic points set of f, namely that the set of weakly almost periodic points totally determines the structure of the measure center. In conclusion, it is very meaningful to research on the set of the weakly almost periodic points and the measure center. Author obtained some results which discussed some problems on the set of weakly almost periodic points [10,11]. In this paper, we further investigate the distributional chaos occurring on the measure center.

Throughout this paper, X denotes a compact metric space with metric d, and let $f : X \to X$ be a continuous map. We denote the set of periodic points, almost periodic points, weakly almost periodic points, recurrent points of f by $P(f),\ A(f),\ W(f),\ R(f)$. And let $x \in X,\ y \in X$ is said to be an $\omega - limit$ point of x, if the sequence $f(x), f^2(x), \ldots$ has a subsequence converging to y. The set of $\omega - limit$ of x is denoted by $\omega(x,f)$.

In this paper, we firstly structure the set of weakly almost periodic points, and prove that the one-sided shift (Σ_N, σ) has an uncountable distributionally scrambled set $S \subset W(\sigma) - A(\sigma)$. Then, we give a sufficient condition for the compact dynamic system (X,f) to exhibit distributional chaos on $M(f)$ via semiconjugacy. At last, we prove that the subshift $\sigma|_{M(f)}$ is distributionally chaotic in sequence.

The main results are stated as follows.

Theorem 1. *Let Σ_N be a one-sided symbolic space, and σ be the shift on Σ_N, then σ has an uncountable distributionally scrambled set $S \subset W(\sigma) - A(\sigma)$ such that $\omega(x,\sigma) = \Sigma_N$ for all $x \in S$.*

Corollary 1. *Let Σ_N be a one-sided symbolic space, and σ be the shift on Σ_N. (X,f) and (Σ_N, σ) are topological dynamic systems. Let $h : X \to \Sigma_N$ be a semiconjugacy between f and σ. If there exists some point $y_0 \in \Sigma_N$ such that $h^{-1}(y_0)$ is a singleton, then there exists an uncountable distributionally scrambled set $D \subset M(f)$ of f.*

Corollary 2. *Let Σ_N be a one-sided symbolic space, and σ be the shift on Σ_N, (Σ_N, σ) is a topological dynamic system, then the subshift $\sigma|_{M(\sigma)}$ is distributionally chaotic in a sequence.*

2. Preliminaries

In this section, some basic concepts and lemmas are introduced. This section is divided into two subsection.

2.1. Some basic concepts

Firstly, we introduce the concept of the measure center. In order to introduce the definition of measure center, we give the following terminologies. Denote the *σ-algebra* of Borel subset of X by $\mathscr{B}(X)$. We shall denote the set of all probability measure with respect to $\mathscr{B}(X)$ by $M(X)$. The set of all elements in $M(X)$ which are invariant under f is denoted by $M(X,f)$. We have $M(X) \supset M(X,f) \neq \emptyset$.

Let $m \in M(X,f)$ and let $X_0 \subset X$. X_0 is called an m-measure-1 Borel set for f if $X_0 \in \mathscr{B}(X)$ and $m(X_0) = 1$. X_0 is called an m-measure-1 set for f if X_0 contains an m-measure-1 Borel set for f. X_0 is called an absolute measure-1 Borel set for f if $X_0 \in \mathscr{B}(X)$ and $m(X_0) = 1$ for all $m \in M(X,f)$. X_0 is called an absolute measure-1 set for f if it is an m-measure-1 set for f for all $m \in M(X,f)$.

Definition 1. A subset of X is called the measure center of f if it is the least f-invariant compact absolute measure-1 set for f.

Denote the measure center of f by $M(f)$.

For determining the structure of measure center, zhou put forward the definition of the weakly almost periodic point.

Definition 2. A point $x \in X$ is called a weakly almost periodic point of f if for any $\varepsilon > 0$ there is an integer $N_\varepsilon > 0$ such that for any $n > 0$

$$\sharp\left(\left\{r : d(x, f^r(x)) < \varepsilon, 0 \leqslant r < nN_\varepsilon\right\}\right) \geqslant n,$$

where $\sharp(\cdot)$ denotes the cardinal number of a set.

Denote the set of all weakly almost periodic points of f by $W(f)$.

Definition 3. A point $x \in X$ is called an almost periodic point of f if for any $\varepsilon > 0$, there is an integer $N \in \mathbb{N}$ such that for any integer $q \geqslant 0$, there is an integer r with $q \leqslant r < q + N$ such that $d(f^r(x), x) < \varepsilon$.

Denote the set of all almost periodic points of f by $A(f)$.

It is not difficult to see that $P(f) \subset A(f) \subset W(f) \subset R(f)$, $f(W(f)) \subset W(f)$, and $M(f) = \overline{W(f)}$ (see the Ref. [1]).

Distributional chaos, which was introduced by Schweizer and Smítal, is based on the asymptotic distance distributions of two trajectories [8].

Let

$$F_{xy}(t) = \liminf_{n \longrightarrow \infty} \frac{1}{n} \sum_{i=1}^{n} \chi_{[0,t)}(d(f^i(x), f^i(y)));$$

$$F^*_{xy}(t) = \limsup_{n \longrightarrow \infty} \frac{1}{n} \sum_{i=1}^{n} \chi_{[0,t)}(d(f^i(x), f^i(y))),$$

where $\chi_A(y)$ is 1 if $y \in A$, and 0 otherwise. Obviously, F_{xy} and F^*_{xy} are both nondecreasing functions. If for $t \leqslant 0$ we define $F_{xy}(t) = F^*_{xy}(t) = 0$, then F_{xy} and F^*_{xy} are probability distributional functions.

Definition 4. Let $D \subset X,\ \forall x,\ y \in D,\ x \neq y$, if we have

(1) $\exists \delta > 0, \quad F_{xy}(\delta) = 0,$

(2) $\forall t > 0, \quad F^*_{xy}(t) = 1,$