Chaos, Solitons & Fractals 45 (2012) 1343–1348

Contents lists available at SciVerse ScienceDirect

Chaos, Solitons & Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Competition analysis of a triopoly game with bounded rationality

A.A. Elsadany

Department of Basic Science, Faculty of Computers and Informatics, Suez Canal University, Ismailia 41522, Egypt

A R T I C L E   I N F O

Article history:
Received 15 June 2011
Accepted 2 July 2012
Available online 21 September 2012

A B S T R A C T

A dynamic Cournot game characterized by three boundedly rational players is modeled by three nonlinear difference equations. The stability of the equilibria of the discrete dynamical system is analyzed. As some parameters of the model are varied, the stability of Nash equilibrium is lost and a complex chaotic behavior occurs. Numerical simulation results show that complex dynamics, such as, bifurcations and chaos are displayed when the value of speed of adjustment is high. The global complexity analysis can help players to take some measures and avoid the collapse of the output dynamic competition game.

1. Introduction

Market economy is essentially a dynamic system, which is mathematically represented by differential or difference equations. In the dynamic theory of economics, there are a lot of differential or difference dynamic models, such as the classical cobweb model describing the variation of the supply and demand, the Cournot models of oligopoly, and so on. Based on these models, the analysis and control of the stable, periodic and chaotic dynamic evolution of the market economy system are investigated, and a series of results have been obtained (see [1]). It is well known that the triopoly game is one of the fundamental oligopoly games. The classic triopoly game of quantity competition is a game between three firms that simultaneously choose quantities, with Cournot's solution as the unique Nash equilibrium. In repeated triopoly games all players maximize their profits. Recently, dynamics of triopoly game has been studied in [2–6]. Bischi and Naimzada [7] gave mathematical specification of a duopoly game with bounded rationality. Agiza et al. [8] examined the dynamical behavior of Bowley's model with bounded rationality. They also have studied the complex dynamics of bounded rationality duopoly game with nonlinear demand function [9]. The triopoly game is closer to the economic reality and is worth being used in oligopoly, but the analysis of dynamic is more complicated. So, a triopoly game with three homogeneous players has been studied by Puu [10]. He considered all players as naive players, and showed that the dynamics of a triopoly game can be complex and that cycles and that chaos may arise. Agiza et al. [4] extended the Kopel duopoly game [11] to the triopoly case, and studied the multistability of the game. The complexity of electric power triopoly game has been studied in [12].

The triopoly with boundedly rational game players that I study in this paper is based on the assumption that the three producers have not a complete knowledge of the market, hence they behave adaptively, following a adjustment process based on a local estimate of the marginal profit $\frac{\partial \pi_i}{\partial q_i}$ [7]. The dynamic adjustment of this repeated Cournot triopoly game can be modeled as

$$q_i(t+1) = q_i(t) + \alpha_i q_i \frac{\partial \pi_i(q_1, q_2, q_3)}{\partial q_i}, \quad i = 1,2,3. \qquad (1)$$

where $\pi_i(q_1,q_2,q_3)$ is the one-period profit of producer i and α_i is a positive parameter which represents the relative speed of production adjustment of producer i. As usual in triopoly models, the price of the good is determined by the total supply $Q(t) = q_1 + q_2 + q_3$ through a given inverse demand function $p = f(Q)$, so that the one-period profit for firm i is given by

$$\pi_i(q_1, q_2, q_3) = q_i f(q_1 + q_2 + q_3) - c_i q_i, \quad i = 1,2,3. \qquad (2)$$

where the positive constants c_i represent the marginal cost per unit of the ith firm. With this assumption, the time evolution of the dynamic game is determined by the iteration of the following three-dimensional map:

E-mail address: aelsadany1@yahoo.com

0960-0779/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.chaos.2012.07.003

$$T:\begin{cases} q'_1 = q_1 + \alpha_1 q_1\left[f(q_1+q_2+q_3) + q_1\frac{\partial f}{\partial q_1} - c_1\right], \\ q'_2 = q_2 + \alpha_2 q_2\left[f(q_1+q_2+q_3) + q_2\frac{\partial f}{\partial q_2} - c_2\right], \\ q'_1 = q_1 + \alpha_3 q_3\left[f(q_1+q_2+q_3) + q_3\frac{\partial f}{\partial q_3} - c_3\right], \end{cases} \quad (3)$$

Starting from some non-negative initial productions $(q_1(0), q_2(0), q_3(0)) = (q_{1,0}, q_{2,0}, q_{3,0})$, the iteration of Eq. (3) is uniquely defined through the forward time evolution of the repeated game, represented by the trajectory $(q_1(t), q_2(t), q_3(t)) = T^t(q_{1,0}, q_{2,0}, q_{3,0})$, $t = 0, 1, 2, \ldots$.

In this paper the modification of demand function introduced by Offerman et al. [13] is applied to study the nonlinear triopoly game considered in Agiza et al. [9], where costs of boundedly rational players were modeled in a linear manner. The main purpose is to study the complex dynamics analysis and control of triopoly game which has nonlinear demand function. We shall study the existence of positive equilibria and then investigate their local stability showing that complex dynamics may emerge. Moreover, the delayed feedback control method is proposed to drive the game to the Nash equilibrium point. This paper is organized as follows. In Section 2, the triopoly game with boundedly rational players is briefly described. In Section 2 the equilibrium points and their stability are studied. Section 3 is devoted to numerical simulations, that confirm analytical results. Conclusions are given in Section 4.

2. The model

We consider three firms, labeled by $i = 1, 2, 3$ producing the same good for sale in the market. Production decisions of all firms are taken at discrete time periods $t = 0, 1, 2, \ldots$. Let $q_i(t)$ denotes the output of ith firm during period t, and a production cost $C_i(q_i)$. The price at period t is determined by the total supply $Q(t) = q_1(t) + q_2(t) + q_3(t)$ through an inverse demand function belonging to the family:

$$p(Q) = a - bQ^n. \quad (4)$$

It is known that for $n = 1$ we have the linear demand function while, for instance, if $a = 0$, $b = -1$ and $n = -1$ we have the isoelastic demand function used by Puu [10] and then in a series of other works [2,3,5,10].

In the present paper we consider $n = 1/2$ as in Offerman et al. [13], obtaining:

$$p(Q) = a - b\sqrt{Q}, \quad (5)$$

where a and b are positive constants. This function is convex as the isoelastic demand function but it does not tend to infinity as $p \to 0$. In fact, $(a/b)^2$ represents the maximum amount of output that can be brought to the market. Those properties are important/relevant from an economic point of view, that is the fact that this demand function belongs to the family of function with the elasticity is fixed. Moreover this form has also used in others oligopoly models and in laboratory experiments economics dealing with learning and expectations formation (see e.g. [9,14–16]). The cost function is linear:

$$C_i(q_i) = c_i q_i, \quad i = 1, 2, 3, \quad (6)$$

where the positive parameters c_i are the marginal costs. With these assumptions, the single profit of the ith firm is given by

$$\Pi_i(q_1, q_2, q_3) = q_i(a - b\sqrt{Q}) - c_i q_i, \quad i = 1, 2, 3 \quad (7)$$

and the marginal profit of ith firm at the point (q_1, q_2, q_3) of the strategy space is

$$\Phi i = \frac{\partial \Pi_i}{\partial q_i} = a - c_i - b\sqrt{Q} - \frac{bq_i}{2\sqrt{Q}}, \quad i = 1, 2, 3. \quad (8)$$

The triopoly game with boundedly rational players has the following three-dimensional nonlinear map $T(q_1, q_2, q_3) \to (q'_1, q'_2, q'_3)$ which is defined by

$$T:\begin{cases} q'_1 = q_1 + \alpha_1 q_1\left(a - c_1 - b\sqrt{Q} - \frac{bq_1}{2\sqrt{Q}}\right), \\ q'_2 = q_2 + \alpha_2 q_2\left(a - c_2 - b\sqrt{Q} - \frac{bq_2}{2\sqrt{Q}}\right), \\ q'_3 = q_3 + \alpha_3 q_3\left(a - c_3 - b\sqrt{Q} - \frac{bq_3}{2\sqrt{Q}}\right). \end{cases} \quad (9)$$

2.1. Equilibrium points and local stability

The equilibrium points of the system (9) are the solutions of the following nonlinear equations:

$$\begin{cases} q_1\left(a - c_1 - b\sqrt{Q} - \frac{bq_1}{2\sqrt{Q}}\right) = 0, \\ q_2\left(a - c_2 - b\sqrt{Q} - \frac{bq_2}{2\sqrt{Q}}\right) = 0, \\ q_3\left(a - c_3 - b\sqrt{Q} - \frac{bq_3}{2\sqrt{Q}}\right) = 0. \end{cases} \quad (10)$$

The system (10) have seven fixed points:

$$E_1 = \left(\frac{4(a-c_1)^2}{9b^2}, 0, 0\right),$$

$$E_2 = \left(0, \frac{4(a-c_2)^2}{9b^2}, 0\right),$$

$$E_3 = \left(0, 0, \frac{4(a-c_3)^2}{9b^2}\right),$$

$$E_4 = \left(\frac{4(2a-c_1-c_2)(a-3c_1+2c_2)}{25b^2}, \frac{4(2a-c_1-c_2)(a-3c_2+2c_1)}{25b^2}, 0\right),$$

$$E_5 = \left(\frac{4(2a-c_1-c_3)(a-3c_1+2c_3)}{25b^2}, 0, \frac{4(2a-c_1-c_3)(a-3c_3+2c_1)}{25b^2}\right),$$

$$E_6 = \left(0, \frac{4(2a-c_2-c_3)(a-3c_2+2c_3)}{25b^2}, \frac{4(2a-c_2-c_3)(a-3c_3+2c_2)}{25b^2}\right),$$

$$E_* = (q_1^*, q_2^*, q_3^*), \quad (11)$$

where