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Chaotic transport of a matter-wave soliton in a biperiodically driven optical superlattice

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ABSTRACT

Under the effective particle approximation, we study the temporal ratchet effect for chaotic transport of a matter-wave soliton consisting of an attractive Bose–Einstein condensate held in a quasi-one-dimensional symmetric optical superlattice with biperiodic driving. It is known that chaos can substitute for disorder in Anderson's scenario [Wimberger S, Krug A, Buchleitner A. Phys Rev Lett 2002;89:263601] and only a higher level of disorder can induce Anderson localization for some special systems [Schwartz T, Bartal G, Fishman S, Segev M. Nature 2007;46:52], and a matter-wave soliton can transit to chaos with high or low probability in a high- or low-chaoticity region [Zhu Q, Hai W, Rong S. Phys Rev E 2009;80:016203]. Here we demonstrate that varying the driving phase to break the time reversal symmetry of the system can increase the size of the high-chaoticity region for low- and moderate-frequency regions. Consequently, the parameter region of the exponential spatial localization increases to the same size, and the low-chaoticity and delocalization region, which includes subregions of the ratchet effect and its inverse effect, correspondingly decreases. The positive dependence of the localization on the driving frequency is also revealed. The results indicate that a high-chaoticity region could replace higher disorder and assists in Anderson localization. From the results we suggest a method for controlling directed motion of a matter-wave soliton by adjusting the driving frequency and amplitude to strengthen or suppress, or even reverse, the temporal ratchet effect.

1. Introduction

In recent decades, directed transport based on the ratchet effect has attracted much attention because of extensive applications in physics, chemistry, and biology [1–3]. The ratchet effect comprises spatial and temporal versions. By the spatial ratchet effect we mean that a symmetrically driven particle experiences drift along the easy spatial ratchet direction in a spatially asymmetric saw-tooth potential. Similarly, the temporal ratchet effect could result in drift along the easy temporal ratchet direction for a spatially symmetric potential and a temporally asymmetric driving [4–8]. The spatiotemporal ratchet effect has been applied to manipulate the quantum tunneling for a single particle in a double well [9,10] and to control the transport properties for different systems [4,11,12], both experimentally and theoretically. However, under some special conditions, an inverse ratchet effect was also observed whereby particles move preferentially along the direction in which the potential barriers are steeper [13,14]. This inverse effect can be useful in different fields, such as the design of artificial ratchet-based devices capable of separating DNA molecules [15]. For different systems, current reversals may be caused by different factors, including a change in the particle number per ratchet period between even and odd [13], dissipation-induced symmetry breaking [7], thermal noise [16], and chaos [17].

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Transport phenomena are very important for many problems in physics. One of the most interesting transport phenomena is Anderson localization, which predicts that an electron may become immobile when it is placed in a disordered crystal [18]. As a direct signature of Anderson localization, the exponentially localized wavefunction of a Bose–Einstein condensate (BEC) was observed experimentally in the presence of controlled disorder created by a laser speckle [19] or quasi-periodic optical lattices [20]. The level of disorder may affect the degree of localization for some special systems. This has been demonstrated in a two-dimensional photonic lattice experiment in which ballistic transport becomes diffusive in the presence of disorder and Anderson localization occurs at a higher level of disorder [21]. Here we are interested in the dependence of chaoticity on the temporal ratchet effect and Anderson localization.

In the mean field theory, BEC is described by the Gross–Pitaevskii equation (GPE). In the absence of an external potential, the GPE describes the well-known bright soliton solution for attractive atomic interactions. Clearly, in the first-order perturbation approximation, a weak external potential may not affect the existence of a bright soliton such that the matter-wave soliton can be considered as an effective classical particle if it is narrow compared to the lattice period [11,22–26]. Using the effective particle approximation (EPA), Poletti et al. studied soliton transport for a BEC held in a weak flashing ratchet potential and found that the average velocity of the soliton depends on its effective mass [11]. According to the degree of symmetry breaking (DSB) mechanism [27,28], Rietmann et al. proposed an analytical estimate for the average velocity [22]. In the presence of periodic driving, the corresponding effective Hamiltonian is associated with a classically chaotic system [23–26,29]. Because chaos sensitively depends on the initial conditions [30] and the initial conditions cannot be set accurately in a real experiment, the average velocity must be used over the initial conditions and times for studying chaotic transport in a deterministic rocking ratchet [31]. It is known that chaos can substitute for disorder in Anderson's scenario [32,33] and only a higher level of disorder can induce Anderson localization for some special systems [21]. In a previous study we used the Melnikov function method to investigate the chaotic region of soliton motion [34]. It has been shown that for a stochastic initial setup a matter-wave soliton can transit to chaos with high or low probability in a high- or low-chaoticity region. In investigations of Anderson localization, whether high chaoticity can replace higher disorder remains an interesting question.

Here we consider a matter-wave soliton consisting of an attractive BEC held in a quasi-1D symmetric optical superlattice without disorder, and use the EPA method and biperiodic driving to study the temporal ratchet effect in chaotic transport. First, we analytically and numerically demonstrate that varying the driving phase to break the time reversal symmetry of the system can increase the size of the high-chaoticity region for low- and moderate-frequency regions. Furthermore, we study chaotic transport of the soliton by numerically calculating the mean square displacement of the solitonic center of mass and the average velocity, defined as the current [31]. We find that an asymmetric temporal ratchet leads to enlarging the same size of the parameter region for dynamic localization [35]. Consequently, the delocalization region, which includes subregions of the ratchet effect and its reversal related to low chaoticity, correspondingly decreases. Numerical results show that the size of the localization region can be further increased by increasing the driving frequency, and the temporal ratchet effect can be enhanced or suppressed, or even reversed, by adjusting the driving parameters. We also show that dynamic localization with zero mean square displacement implies exponential spatial localization, which is directly related to Anderson localization [19,20]. We demonstrate that high chaoticity can substitute for higher disorder in Anderson localization, and suggest a useful method for controlling the directed motion of a matter-wave soliton by applying the ratchet effect.

2. Asymmetry increases the size of the high-chaoticity region

The system considered here is a bright matter-wave soliton consisting of an attractive BEC held in a quasi-1D symmetric optical superlattice with biperiodic driving. The soliton dynamics is governed by the 1D GPE [23–25]

$$i\psi_t + \psi_{xx} + 2\psi|\psi|^2 = V\psi, \tag{1}$$

where the external potential is

$$\begin{aligned} V &= V_0[\alpha\cos x + (1-\alpha)\cos(2x)] + f(t)x, \\ f(t) &= f_0[\eta\cos(\omega t) + (1-\eta)\cos(2\omega t + \phi))] \end{aligned} \tag{2}$$

without disorder. The parameters $V_0\alpha$ and $V_0(1-\alpha)$ are the weak potential depth of the primary and secondary lattices, respectively. The driven linear potential is $f(t)x$, where $f(t)$ is the weak driving force of small intensity f_0. η and ϕ denote the relative amplitude and phase difference, respectively, of the double cosine driving. A similar linear potential has been applied in the ultracold atomic system [36] and the nonlinear Schrodinger equation [37]. To simplify, we adopted dimensionless variables and parameters [11]: energy, length, frequency, and time are measured in units of $E_r = \hbar^2k^2/(2m)$, k^{-1}, $\omega_0 = \hbar k^2/(2m)$ and ω_0^{-1}, respectively, where k is the wave vector of the optical lattice and m is the atomic mass. Thus, the normalized wave function in Eq. (1) can be written as [11] $\psi = \psi_{1D}\sqrt{g_{1D}}$, where ψ_{1D} is the original 1D wave function, $g_{1D} = 2ka_s\omega_r/\omega_0$ is the interatomic interaction strength, a_s is the s-wave scattering length, and ω_r is the radial trap frequency. The number of atoms in the system is given by $N_0 = N/g_{1D} = \int|\psi|^2dx/g_{1D}$. For a given atomic number N_0, N is adjusted by g_{1D}. The parameter α is fixed and the value $\eta \in [0,1]$ is adopted throughout the paper.

Obviously, the time-inversion symmetry is broken if $\phi \neq n\pi$ for $n = 0,1,2,\ldots$, which can allow directed transport of matter-wave solitons [38]. Taking the parameters $\omega = 2.1$ and $\eta = 2/3$, the asymmetric relative driving force $f(t)/f_0$ is plotted in Fig. 1 for (a) $\phi = \pi/2$ and (b) $\phi = -\pi/2$. It is clear the easy ratchet directions are (a) to the left and (b) to the right. The temporal ratchet effect leads to