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Bifurcation structure of chaotic attractor in switched dynamical systems with spike noise

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ABSTRACT

High-frequency ripple (spike noise) effects in the qualitative properties of DC/DC converter circuits. This study investigates the bifurcation structure of a chaotic attractor in a switched dynamical system with spike noise. First, we introduce the system dynamics and derive the associated Poincaré map. Next, we show the bifurcation structure of the chaotic attractor in a system with spike noise. Finally, we investigate the dynamical effect of spike noise in the existence region of the chaotic attractor compare with that of a chaotic attractor in a system with ideal switching. The results suggest that spike noise enlarges an invariant set and generates a new bifurcation structure of the chaotic attractor.

1. Introduction

DC/DC converters are a typical example of power electronic circuits and are categorized as switched dynamical systems (SDSs). These circuits exhibit rich and interesting dynamics including subharmonics, quasi-periodic oscillations, and chaotic behavior [1–6]. It is important to examine these dynamics from a practical viewpoint because they are common in many engineering fields.

Bifurcation phenomena in piecewise smooth maps are of significant interest. The discrete map of a switching converters is generally a piecewise smooth map. Thus, it is important to analyze the fundamental characteristics of piecewise smooth maps from a mathematical viewpoint [7–11]. In particular, nonlinear phenomena in one-dimensional piecewise smooth maps is worthy of analysis [12–15] because switching converters sometimes construct one-dimensional piecewise smooth maps [16–19]. Many researchers have studied the nonlinear phenomena in such maps for the existence region of not only periodic solutions but also chaotic attractor in order to understand the relevant fundamental characteristics [8,20–24].

Until now, nonlinear phenomena in switching converters have been studied under the assumption of ideal switching action. However, Ref. [25] reports the experimental appearance of high-frequency ripple (spike noise) effects in the fundamental characteristics of current-mode controlled DC/DC converters. Using a simplified one-dimensional model, Ref. [25] mathematically clarified that a new mapping region appears in the piecewise smooth map of a converter circuit with spike noise. However, beyond this report, the dynamical effect of spike noise in piecewise smooth maps has not otherwise been reported. Therefore, we have previously proposed a one-dimensional SDS, whose discrete model has the same piecewise smooth characteristics as those described in Ref. [25], and presented a detailed analysis of the existence region of the periodic solution [26]. However, the dynamical effect of spike noise in the existence region of a chaotic attractor has not yet been studied. In addition, there is no paper to clarify how the new mapping region in the piecewise smooth map of a SDS with spike noise influences the existence region of chaotic attractor.

The main purpose of this study is to examine the dynamical structure of a piecewise smooth map with spike noise in the existence region of a chaotic attractor as well as the attractor's bifurcation structure. On the basis of Ref. [25], we assume that a suitable amount of spike noise

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occurs in SDS. First, we introduce the system dynamics and derive the associated Poincaré map with piecewise smooth characteristics. Next, we clarify the bifurcation structure of the chaotic attractor in a system with spike noise by using the composite Poincaré map. Finally, we discuss the dynamical effect of spike noise in the existence region of the chaotic attractor compare with that of chaotic attractor in a system with ideal switching. We conclude that the new mapping region in the piecewise smooth map with spike noise generates the new bifurcation structure of the existence region of the chaotic attractor.

2. Switched dynamical system

2.1. System description

Consider the following SDS.

$$\frac{dx}{dt} = -x + H(t,x), \tag{1}$$

where $H(t,x)$ is defined as follows:

$$H(t,x) = \begin{cases} B, & x < X_r, \quad \text{for state-1}, \\ 0, & t < kT, \quad \text{for state-2}. \end{cases} \tag{2}$$

In Eq. (1), x is a state variable and t is a time variable. In Eq. (2), $H(t,x)$ is a hysteresis characteristic, X_r is a reference value, and T is the clock interval; hence kT corresponds to the kth clock after the clock interval from $t = 0$. Thus, Eq. (1) can be rewritten as follows:

$$x(t) = \begin{cases} (x_k - B)e^{-(t-kT)} + B, & \text{for state-1}, \\ x_k e^{-(t-kT)}, & \text{for state-2}, \end{cases} \tag{3}$$

where x_k denotes the solution at $t = kT$. Note that we set the parameter B as 3.0 in the following analysis.

Fig. 1 (a) shows the trajectory in a system with ideal switching. When trajectory x reaches the reference value X_r, the system switches from state-1 to state-2. When the clock pulse arrives, one of the following: if the system is in state-2, it returns to state-1; if it is in state-1, it remains at that state.

Fig. 1 (b) shows the trajectory in a system with spike noise. We assume that spike noise is present in all switching actions. The duration of spike noise does not significantly influence the system dynamics [25] and thus is ignored in the following analysis. If spike noise h exceeds the reference value ($x_k + h \geqslant X_r$), the system switches from state-1 to state-2, where it remains until the next clock pulse arrives.

2.2. Poincaré map

Now, we derive the Poincaré map for a system with spike noise. Here, we provide the simplest possible definition of the Poincaré map because we have already explained it in detail in Ref. [26].

First, to classify the trajectory during the clock interval in a system with spike noise, we define borders D_1 and D_2 as follows:

$$D_1 = (X_r - B)e^T + B, \quad D_2 = X_r - h, \tag{4}$$

where we assume that $D_1 < D_2$. Therefore, the regions in which the classified trajectories are observed are defined as follows:

$$\begin{aligned} J_1 &= \{y_k, x_k | x_k \leqslant D_1\}, \\ J_2 &= \{y_k, x_k | (D_1 < x_k < D_2) \text{ or } (D_2 \leqslant x_k \text{ and } y_k < D_1)\}, \\ J_3 &= \{y_k, x_k | D_2 < x_k \text{ and } D_1 \leqslant y_k\}, \end{aligned} \tag{5}$$

where if x_k is the solution at the kth clock, we define y_k as the solution at the $(k-1)$th clock. Note that a solution y_k

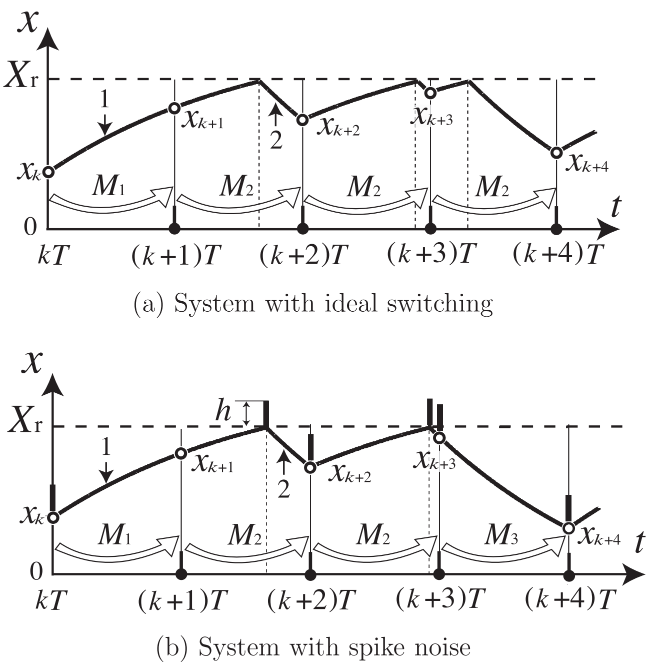

Fig. 1. Conceptual trajectory diagrams.