Chaos, Solitons & Fractals 45 (2012) 899–908

Contents lists available at SciVerse ScienceDirect

Chaos, Solitons & Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

An adaptive stochastic model for financial markets

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ARTICLE INFO

Article history:
Received 16 November 2011
Accepted 9 March 2012
Available online 5 April 2012

ABSTRACT

An adaptive stochastic model is introduced to simulate the behavior of real asset markets. The model adapts itself by changing its parameters automatically on the basis of the recent historical data. The basic idea underlying the model is that a random variable uniformly distributed within an interval with variable extremes can replicate the histograms of asset returns. These extremes are calculated according to the arrival of new market information. This adaptive model is applied to the daily returns of three well-known indices: Ibex35, Dow Jones and Nikkei, for three complete years. The model reproduces the histograms of the studied indices as well as their autocorrelation structures. It produces the same fat tails and the same power laws, with exactly the same exponents, as in the real indices. In addition, the model shows a great adaptation capability, anticipating the volatility evolution and showing the same volatility clusters observed in the assets. This approach provides a novel way to model asset markets with internal dynamics which changes quickly with time, making it impossible to define a fixed model to fit the empirical observations.

1. Introduction

A huge number of works aimed at describing the financial market dynamics have been presented over the last decades. Although many models have been proposed, using different approaches, they usually can only reproduce a subsample of the characteristics observed in real market. The complexity of market dynamics has led the scientific community to divide the problem into several lines of research, such as the study of the probability distribution of the asset returns or the mechanisms governing why such returns show certain autocorrelations. Hence, many aspects of financial markets remain undiscovered and new models should be proposed in order to complete its understanding.

From a statistical point of view, several probability distributions have been introduced as candidates to fit the asset returns histograms of financial markets: Lévy [1], Normal Inverse Gaussian [2] and KR [3] distributions among others. Besides that, other probability distributions have been proposed for internal processes related to price formation which might induce the heavy tailed distributions of asset returns. Such is the case of the student distribution assigned to the order placement process in the order book [4,5]. An important problem arises when trying a theoretical probability distribution to fit with an asset returns histogram, since the latter usually shows fat tails allocating excessive probability to large returns. This mismatch with the theoretical tails led to one of the most active lines of research: the search for more suitable distributions and the underlying mechanisms which induce those empirical large tails [6,7]. In this direction an important conclusion is that the empirical histograms seem to be described by power laws with very similar exponents for different types of markets [8,9].

On the other hand there are additional facts which cannot be explained from a statistical point of view. While the Autocorrelation Function of returns decays rapidly with time, the Autocorrelation Function of absolute returns remains significant indicating positive autocorrelation [7], which has been related to another well-known effect: volatility clustering [7,10]. It consists in bursts of high volatility located within precise time intervals, diverging

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http://dx.doi.org/10.1016/j.chaos.2012.03.005

considerably from the average behavior. Volatility clusters are a typical characteristic of financial markets.

Many models, with different approaches, have been proposed with the aim of reproducing some of the previous characteristics in order to identify the basic precursor mechanisms for them. Next, a general review of the most important kinds of models is shown.

A relevant group of models is the so-called ''Agent models'', which attempt to explain a financial market as an aggregation of single agents, or clusters of agents, with specific roles assigned. Every role would be a basic investment behavior. A good overview of agent models can be found in [11]. While belonging to the same group, a variety of different models have been introduced, depending on the roles assigned to the involved agents [12–14]. Some of these works are focused on how an agent-based model can reproduce some real market characteristics [15,16]. It is worth mentioning the so-called ''herd behavior'' i.e. of the appearance of imitation patterns among the investors which might be responsible for the observed autocorrelation in some periods of an asset evolution [17–19]. Although herd behavior is usually related to agent-based models other types of models may incorporate this concept.

The order-book models form another active line of research. The order-book, in a rough view, is a computing system which gathers all the incoming sell and buy limit orders, placing them along a price axis according to their price targets. Along with these limit orders, market and cancellation orders are already introduced in the book. The former consist in orders to be matched with the best price at the moment, while the latter are cancellations of previous limit orders. The interactions among all these orders constitute the basic dynamics of price formation. Thereby the models of the order-book aim to capture the essence of how the price of an asset is built [4,20–22].

Modeling by means of stochastic processes is a natural approach, since the asset returns evolution seems to be a random result of the combination of the decisions of many thousands of investors. Several works have introduced stochastic models [23,24]. As commented previously, some phenomena related to autocorrelation have been found in financial markets. Hence, a pure stochastic process without inducing any degree of autocorrelation would not be realistic enough. So, these stochastic models are usually tested on whether they reproduce those phenomena of the real markets.

In addition to those models grouped together in the previous paragraphs, other approaches have been proposed such as the ''microscopic spin model'' [25] or models based on the ''quantum field theory'' [26], among others.

This work introduces an adaptive stochastic model which can change its parameters endogenously by using the recent historical data, in order to assimilate the new information incorporated in an asset market. The basic idea underlying the model is that the multiplication of two different uniform probability distributions can replicate any empirical histogram when the parameters of both distributions are fitted appropriately. The idea of this multiplication of two uniform distributions resulted from a general Dynamics of Resource Density which is also presented in this work.

In one sense, the Dynamics of Resource Density is a macroscopic view of the order-book state over a time interval, after aggregating all the single orders received along such an interval. However, it is not a microscopic view of how the processes interact in the order-book, but a sequence of macroscopic equivalent states resulted from the combination of the microscopic processes. So, the Dynamics of Resource Density is not an order-book model, but a high level model of the evolution of supply and demand.

The first goal of this work is to demonstrate that a general simulator of probability distributions is derived from a generic Dynamics of Resource Density, consisting in the multiplication of two uniform distributions with parameters depending on the specific system under study. The second goal is to build an adaptive stochastic process with the previous simulator, fitting automatically its parameters according to the historical evolution of the asset.

This model is applied in the paper to three well-known indices: the Spanish Ibex35, the American Dow Jones and the Japanese Nikkei, thereby covering European, American and Asian markets. The time series used are the daily returns of every index from 2008 to 2010. It is found that the histograms produced by the model and the real ones are really close, showing the same fat tails and power laws. Moreover, the model replicates correctly the real autocorrelation structure and volatility clustering.

The work is organized as follows. The Dynamics of Resource Density is presented in Section 2, introducing the dynamics itself and obtaining a general probability simulator, based on the multiplication of two uniform distributions. In the same section it is demonstrated that the previous multiplication is equivalent to only one uniform distribution defined in an interval with random limits. The adaptive stochastic model is introduced in Section 3. Applications of the model to Ibex35, Dow Jones and Nikkei indices are shown in Section 4. Finally, the conclusions of the work are summarized in Section 5.

2. Dynamics of Resource Density

The dynamics of a distributed resource with border constraints, affected by an external demand, is studied in this section. This is a specific case of the most general situation in which a distributed resource evolves when external demands can take place in any part of its distribution. An example of the general case is a predator feeding on a specimen in an ecosystem. Here, the resource density is the distribution of the specimen within the ecosystem, that is a function of two or three dimensions, depending of the ecosystem in question. The external demand is the predator's need for feeding, inducing a resource consumption within the whole surface or volume of the ecosystem. In this example there is not any kind of constraint, since the resource can be consumed in any part of the ecosystem. On the other hand, the growth processes can be represented as resource densities with border constraints. For example, the tumoral growth in a patient is a process in which the tumoral cells, belonging to the border of the tumor, demand space in the