Available online at www.sciencedirect.com
ScienceDirect
Chaos, Solitons and Fractals 40 (2009) 1688–1696
CHAOS SOLITONS & FRACTALS
www.elsevier.com/locate/chaos

LMI conditions for stability of stochastic recurrent neural networks with distributed delays ☆

R. Rakkiyappan, P. Balasubramaniam *

Department of Mathematics, Gandhigram Rural University, Gandhigram 624 302, Tamil Nadu, India

Accepted 13 September 2007

Communicated by Prof. Ji-Huan He

Abstract

In this paper, the global asymptotic stability of stochastic recurrent neural networks with discrete and distributed delays is analyzed by utilizing the Lyapunov–Krasovskii functional and combining with the linear matrix inequality (LMI) approach. A new sufficient condition ensuring the global asymptotic stability for delayed recurrent neural networks is obtained in the stochastic sense using the powerful MATLAB LMI Toolbox. In addition, an example is also provided to illustrate the applicability of the result.

1. Introduction

In the past two decades, different classes of neural networks with or without time delays, including Cohen–Grossberg neural networks [1], Cellular neural networks [2] and bidirectional associative memory networks [3] have been discussed. Although most neural systems are realized by software simulations, only hardware implementation can fully utilize its advantages of parallel processing and error tolerance. However, these successful applications are greatly dependent on the dynamic behaviors of neural networks. It is well known that stability is one of the main properties for a crucial feature in the design of neural networks. The time delays are commonly encountered in various engineering systems such as chemical processes, hydraulic and rolling mill systems, etc. These effects are unavoidably encountered in the implementation of neural networks, and may cause undesirable dynamic network behaviors such as oscillation and instability. Therefore, it is important to investigate the stability of delayed neural networks. A large number of the criteria on the stability of neural networks have been derived in the literatures [4–11].

So far, most works on delayed neural networks have dealt with the stability analysis problems for neural networks with discrete delays. Neural networks has a spatial nature due to the presence of parallel pathways with a variety of axon sizes and lengths. So it is desirable to model them by introducing unbounded delays. In recent years there has been a growing research interest in the study of neural networks with distributed delays. In fact, both discrete and

☆ The work of the authors was supported by UGC-SAP(DRS), New Delhi, India under the sanctioned No. F510/6/DRS/2004.
* Corresponding author. Tel.: +91 4512452371; fax: +91 4512453071.
E-mail address: pbalgri@rediffmail.com (P. Balasubramaniam).

0960-0779/$ - see front matter © 2007 Published by Elsevier Ltd.
doi:10.1016/j.chaos.2007.09.052

distributed delays should be taken into account when a modeling a realistic neural network [12–15]. It should be mentioned that using linear matrix inequality (LMI) approach the sufficient global asymptotic stability conditions have been derived in [16,17] for a general class of neural networks with both discrete and distributed delays. Very recently, Zhang et al. [18] studied global exponential stability for nonautonomous cellular neural networks with unbounded delays. So far, there are only a few papers that have taken stochastic phenomenon into account in neural networks [19–21]. Practically, such phenomenon always appears in the electrical circuit design of neural networks. Wang et al. [22,23] were studied the exponential stability of uncertain stochastic neural networks with discrete and distributed delays and robust stability for stochastic Hopfield neural networks with time delays.

Based on the above discussions, we consider a class of stochastic neural networks with unbounded distributed delays. The main purpose of this paper is to study the global asymptotic stability for stochastic neural networks with unbounded distributed delays. To the best of the authors' knowledge there were no global stability results for stochastic recurrent neural networks with unbounded distributed delays. By using Lyapunov–Krasovskii functional we obtain the sufficient conditions for global asymptotic stability of stochastic recurrent neural networks, in terms of linear matrix inequality (LMI), which can be easily calculated by MATLAB LMI toolbox. We also provide two numerical examples to demonstrate the effectiveness of the proposed stability results.

2. Global stability results

Throughout the manuscript we will use the notation $A > 0$ (or $A < 0$) to denote that the matrix A is a symmetric and positive definite (or negative definite) matrix. The notation A^{T} and A^{-1} mean the transpose of A and the inverse of a square matrix. If A, B are symmetric matrices $A > B$ ($A \geqslant B$) means that $A - B$ is positive definite (positive semi-definite).

Consider neural network with discrete and distributed delays can be described by the following integro-differential equations

$$x_i'(t) = -a_i x_i(t) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} c_{ij} f_j(x_j(t-\tau_j(t))) + \sum_{j=1}^{n} d_{ij} \int_{-\infty}^{t} k_j(t-s) f_j(x_j(s))\,\mathrm{d}s + I_i, \tag{1}$$

$i = 1, 2, \ldots, n$, where $x_i(t)$ is the state of the ith neuron at time t, $a_i > 0$ denotes the passive decay rate, b_{ij}, c_{ij} and d_{ij} are the synaptic connection strengths, f_j denotes the neuron activations, I_i is the constant input from outside the system, $\tau(t)$ represents the discrete transmission delay with $\dot{\tau}(t) \leqslant \eta < 1$ and the delay kernel k_j is a real valued continuous function defined on $[0, +\infty]$ and satisfies, for each i,

$$\int_{0}^{\infty} k_j(s)\,\mathrm{d}s = 1. \tag{2}$$

We assume that the neuron activation functions f_j, $j = 1, 2, \ldots, n$ satisfy the following hypotheses, respectively:

(H_1) f_j is bounded function for any $j = 1, 2, \ldots, n$.
(H_2) $0 \leqslant |f_j(\zeta_1) - f_j(\zeta_2)| \leqslant L_j|\zeta_1 - \zeta_2|$ for all $\zeta_1, \zeta_2 \in R$, $\zeta_1 \neq \zeta_2$.

Assume $x^* = (x_1^*, x_2^*, x_n^*)^{\mathrm{T}}$ is an equilibrium point (1), one can derive from (1) that the transformation $y_i = x_i - x_i^*$ transforms system (1) into the following system:

$$y'(t) = -Ay(t) + Bg(y(t)) + Cg(y(t-\tau(t))) + D\int_{-\infty}^{t} K(t-s) g(y(s))\,\mathrm{d}s \tag{3}$$

where $y = [y_1, y_2, \ldots, y_n]^{\mathrm{T}}$, $A = \mathrm{diag}[a_1, a_2, \ldots, a_n]$, $B = [b_{ij}]$, $C = [c_{ij}]$, $D = [d_{ij}]$.

$K(t-s) = \mathrm{diag}[k_1(t-s), k_2(t-s), \ldots, k_n(t-s)]$, $g(y) = [g_1(y_1), g_2(y_2), \ldots, g_n(y_n)]^{\mathrm{T}}$ with $g_j(y_j(t)) = f_j(y_j(t) + x_j^*) - f_j(x_j^*)$. Note that since each function $f_j(\cdot)$ satisfies the hypotheses (H_1)–(H_2), hence each $g_j(\cdot)$ satisfies

$$g_j^2(\zeta_j) \leqslant L_j^2 \zeta_j^2, \quad \zeta_j g_j(\zeta_j) \geqslant \frac{g_j^2(\zeta_j)}{L_j}, \quad \forall \zeta_j \in R, \quad g_j(0) = 0.$$

Consider the following stochastic delayed recurrent neural networks with time varying delay is described by

$$\mathrm{d}y(t) = \left[-Ay(t) + Bg(y(t)) + Cg(y(t-\tau(t))) + D\int_{-\infty}^{t} K(t-s) g(y(s))\,\mathrm{d}s\right]\mathrm{d}t + \sigma(t, y(t), y(t-\tau(t)))\,\mathrm{d}w(t) \tag{4}$$