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Waves in three-dimensional simple cubic lattice

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Abstract

Linear and nonlinear vibrations of particles in a three-dimensional (3D) simple cubic lattice are investigated in this paper. The linear wave equations are derived and the dispersion relations of both longitudinal and transverse waves in the different directions are given analytically. Furthermore, the nonlinear waves in this lattice have also been studied. Solitary waves in several special directions have been investigated as well.

1. Introduction

The waves in one-dimensional (1D) and/or two-dimensional (2D) lattice consisting of charged particles have been studied extensively. These systems contain particles which arrange themselves in a crystalline structure in the presence of external and inter-particle forces. As well known, the first important investigation of the nonlinear lattice was probably in the 1950s with the numerical work by Fermi, Pasta, and Ulam (FPU) [1]. Theirs work was one of those cases where numerical computations revealed quite unexpected results which led to the synergetic enhancement of understanding through combined computational and analytical work. Later, Toda proposed a nonlinear lattice with a nearest neighbor interaction potential, and opened a door for a better understanding of nonlinear phenomena, not only the FPU recurrence, but also those in other fields related to the nonlinear mechanics [2,3]. Wadati and Toda have done much work on nonlinear waves in the lattice. As we know, the famous Korteweg de Vries (KdV) equation has become one of the most important nonlinear equation to study solitons. It describes not only the shallow water waves, but also hydromagnetics waves in cold plasma, ion acoustic waves (IAW), dust acoustic waves (DAW), and dust ion acoustic waves (DIAW) [4–6]. Ilzuka and Wadati have studied the nonlinear waves in lattice with some discontinuities [7–9]. They assume that there are N same particles which are all in one direction. Each particle interacts only with the nearest neighbors. For different interaction potential between two particles, the properties of nonlinear waves can be different. If the forces are linear, it is easier to give the analytical solution of the lattice. However, if the forces are nonlinear, it is difficult to know the analytical solution. Then, we have to use either the reductive perturbation technique or the numerical method to find the answers. These kinds of works have already been done. For example, Tsai studied the solutions of nonlinear lattice, in which the nonlinear potential are cubic and quadratic [10,11]. There are many researches on nonlinear lattices of Morse potential and Lennard-Jones (L-J) potential as well [12,13].

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Nowadays, there are more and more researchers who are studying the dusty plasmas. An important feature for it is the existence of structures in the strongly coupled dusty plasmas. In a plasma, there is an important parameter, usually expressed by Γ, which determine whether the plasma is strongly coupled or not. Γ is the ratio of the potential energy to the kinetic energy of the particles. It is generally considered as the strongly coupled when $\Gamma > 1$. When $\Gamma > \Gamma_c$, where Γ_c is critical value, there is a phase transition to solid state and a Coulomb lattice is formed. In the strong coupling regime, plasma display a solid-like behavior which have been observed and studied in many laboratory experiments [14–16]. The linear and nonlinear wave in the dust lattice were studied by many scientists [17–22]. Melandsø [19] presented linear and nonlinear theories of dust lattice wave, taking into account the Yukawa potential among the nearest neighbor particles. As well known, dust lattice sustain longitudinal waves and transverse waves which have been observed and studied in many experiments [23–26]. Furthermore, nonlinear waves in a two-dimensional plasma crystal were observed [27]. As the best of our knowledge, it seems that all the previous work mentioned above are limited to the 1D and 2D lattice. However, the investigations for 3D lattice are too less until now.

Nowadays there are many papers on homotopy perturbation method [28–32] and modified Lindstedt–Poincare methods [33–36]. In this paper, by using the reductive perturbation method, we investigate the linear and nonlinear wave in a 3D simple cubic lattice theoretically. In order to find wave in the lattice we only consider the forces among the nearest particles. The particles interact each other through Coulomb repulsion. The inter-particle spacing is expressed by a which is usually much larger than the wavelength. It is usually sufficient to take into account the "nearest neighbor" and "next nearest neighbor" interactions. For small amplitude waves, the amplitude is much smaller than the inter-particle spacing. Therefore, the displacement from the equilibrium position is relatively small enough.

2. Equation of motion

Now, we consider the 3D simple cubic lattice as shown in Fig. 1. We assume that there is elastic force between two arbitrary particles. In order to find the equation of motion for the (n,m,p)th particle, we only consider the forces exerted on the particle (n,m,p)th by 18 particles. The six nearest neighbor particles are marked by cyan which are expressed as follows: $(n+1,m,p)$th, $(n-1,m,p)$th, $(n,m+1,p)$th, $(n,m-1,p)$th, $(n,m,p+1)$th and $(n,m,p-1)$th, respectively. Set up coordinates as shown in Fig. 1, let the (n,m,p)th particle in equilibrium is in the origin, then the positions of the six neighbor nearest particles at equilibrium are $(a,0,0)$, $(-a,0,0)$, $(0,a,0)$, $(0,-a,0)$, $(0,0,a)$, $(0,0,-a)$, respectively. Similarly, there are 12 next nearest neighbor particles which are marked by red. Their positions are expressed as follows: $(n+1,m,p+1)\text{th} \to (a,0,a)$, $(n,m+1,p+1)\text{th} \to (0,a,a)$, $(n-1,m,p+1)\text{th} \to (-a,0,a)$, $(n,m-1,p+1)\text{th} \to (0,-a,a)$, $(n+1,m+1,p)\text{th} \to (a,a,0)$, $(n-1,m+1,p)\text{th} \to (-a,a,0)$, $(n-1,m-1,p)\text{th} \to (-a,-a,0)$, $(n+1,$

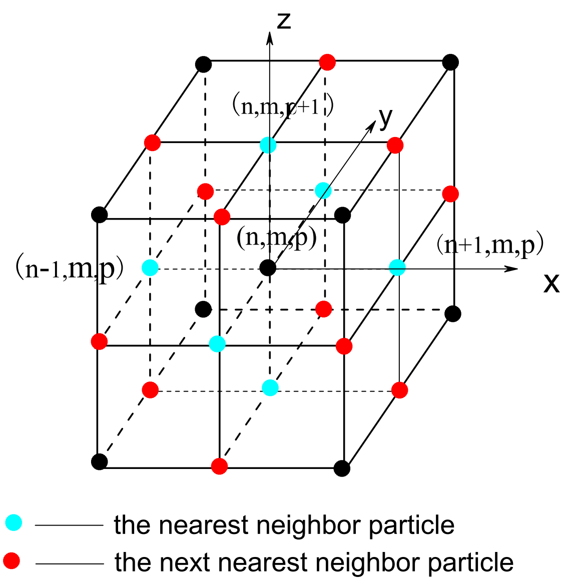

Fig. 1. Model of 3D simple cubic lattice.