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Chaos, Solitons and Fractals 28 (2006) 902–905
CHAOS SOLITONS & FRACTALS
www.elsevier.com/locate/chaos

A note on intuitionistic fuzzy metric spaces ☆

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Accepted 15 August 2005

Abstract

Recently, J.H. Park [J.H. Park, Intuitionistic fuzzy metric spaces. Chaos, Solitons & Fractals 2004;22:1039–46] introduced and studied a notion of intuitionistic fuzzy metric space by using the idea of intuitionistic fuzzy set due to Atanassov. In this note we show that for each intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$, the topology generated by the intuitionistic fuzzy metric (M, N) coincides with the topology generated by the fuzzy metric M, and hence, the study of the space $(X, M, N, *, \diamond)$ reduces to the study of the fuzzy metric space $(X, M, *)$; so that, Park's results follow directly from well-known theorems in fuzzy metric spaces.

1. Introduction and preliminaries

Motivated by the potential applicability of fuzzy topology to quantum particle physics, particularly in connection with both string and $\epsilon^{(\infty)}$ theory developed by El Naschie [2,3], Park introduced and discussed in [14] a notion of intuitionistic fuzzy metric space which is based both on the idea of intuitionistic fuzzy set due to Atanassov [1], and the concept of a fuzzy metric space given by George and Veeramani in [7]. Actually, Park's notion is useful in modelling some phenomena where it is necessary to study the relationship between two probability functions as we will observe in Section 2; for instance, it has a direct physic motivation in the context of the two-slit experiment as the foundation of E-infinity of high energy physics, recently studied by El Naschie in [4,5].

In this note, we shall prove, among other results, that the topology $\tau_{(M,N)}$ generated by an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ coincides with the topology τ_M generated by the fuzzy metric space $(X, M, *)$, and thus, the results obtained in [14] are immediate consequences of the corresponding and well-known results for fuzzy metric spaces.

Let us recall (see [16]) that a *continuous t-norm* is a binary operation $* : [0,1] \times [0,1] \to [0,1]$ which satisfies the following conditions: (i) $*$ is associative and commutative; (ii) $*$ is continuous; (iii) $a * 1 = a$ for every $a \in [0,1]$; (iv) $a * b \leqslant c * d$ whenever $a \leqslant c$ and $b \leqslant d$, and $a, b, c, d \in [0,1]$.

☆ The two first listed authors acknowledge the support of the Plan Nacional I+D+I and FEDER, grant BFM2003-02302.
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0960-0779/$ - see front matter
doi:10.1016/j.chaos.2005.08.113

Similarly, a *continuous t-conorm* is a binary operation $\diamondsuit : [0,1] \times [0,1] \to [0,1]$ which satisfies the following conditions: (i) $\diamondsuit$ is associative and commutative; (ii) $\diamondsuit$ is continuous; (iii) $a \diamondsuit 0 = a$ for every $a \in [0,1]$; (iv) $a \diamondsuit b \leqslant c \diamondsuit d$ whenever $a \leqslant c$ and $b \leqslant d$, and $a, b, c, d \in [0,1]$.

Definition 1. [7] A *fuzzy metric space* is a triple $(X, M, *)$ such that X is a (nonempty) set, $*$ is a continuous t-norm and M is a fuzzy set on $X \times X \times (0,\infty)$ satisfying the following conditions, for all $x, y, z \in X$, $s, t > 0$:

(i) $M(x,y,t) > 0$;
(ii) $M(x,y,t) = 1$ if and only if $x = y$;
(iii) $M(x,y,t) = M(y,x,t)$;
(iv) $M(x,y,t) * M(y,z,s) \leqslant M(x,z,t+s)$;
(v) $M(x,y,\_) : (0,\infty) \to (0,1]$ is continuous.

If $(X, M, *)$ is a fuzzy metric space, we will say that $(M, *)$ (or simply M) is a *fuzzy metric* on X.

Our basic reference for general topology is [6].

George and Veeramani proved in [7] that every fuzzy metric $(M, *)$ on X generates a Hausdorff first countable topology τ_M on X which has as a base the family of open sets of the form $\{B_M(x,r,t) : x \in X,\ r \in (0,1),\ t > 0\}$, where $B_M(x,r,t) = \{y \in X : M(x,y,t) > 1 - r\}$ for all $x \in X$, $r \in (0,1)$ and $t > 0$.

Actually, it is possible to prove the following result.

Theorem 1. [8,10,13] *Let $(X, M, *)$ be a fuzzy metric space. Then (X, τ_M) is a metrizable topological space.*

Further results on the theory of fuzzy metric spaces in the sense of [7] may be found in [7,9,11,12,15].

2. Intuitionistic fuzzy metric spaces

Recently, Park introduced in [14] the following notion:

Definition 2. An *intuitionistic fuzzy metric space* is a 5-tuple $(X, M, N, *, \diamondsuit)$ such that X is a (nonempty) set, $*$ is a continuous t-norm, $\diamondsuit$ is a continuous t-conorm and M, N are fuzzy sets on $X \times X \times (0,\infty)$ satisfying the following conditions, for all $x, y, z \in X$, $s, t > 0$:

(a) $M(x,y,t) + N(x,y,t) \leqslant 1$;
(b) $M(x,y,t) > 0$;
(c) $M(x,y,t) = 1$ if and only if $x = y$;
(d) $M(x,y,t) = M(y,x,t)$;
(e) $M(x,y,t) * M(y,z,s) \leqslant M(x,z,t+s)$;
(f) $M(x,y,\_) : (0,\infty) \to (0,1]$ is continuous;
(g) $N(x,y,t) < 1$;
(h) $N(x,y,t) = 0$ if and only if $x = y$;
(i) $N(x,y,t) = N(y,x,t)$;
(j) $N(x,y,t) \diamondsuit N(y,z,s) \geqslant N(x,z,t+s)$;
(k) $N(x,y,\_) : (0,\infty) \to [0,1)$ is continuous.

If $(X, M, N, *, \diamondsuit)$ is an intuitionistic fuzzy metric space, we will say that $(M, N, *, \diamondsuit)$ (or simply (M, N)) is an *intuitionistic fuzzy metric on X.*

Remark 1. (A) Observe that condition (g) of Definition 2 follows directly from (a) and (b).

(B) It is clear that if $(X, M, N, *, \diamondsuit)$ is an intuitionistic fuzzy metric space, then $(X, M, *)$ is a fuzzy metric space in the sense of Definition 1. Conversely, if $(X, M, *)$ is a fuzzy metric space, then $(X, M, 1 - M, *, \diamondsuit)$ is an intuitionistic fuzzy metric space, where $a \diamondsuit b = 1 - [(1 - a) * (1 - b)]$ for all $a, b \in [0,1]$.

In the analysis of the probability involved in the two-slit experiment [4], denote by $P_1(x)$ the probability that a particle x pass through slit 1 and by $P_2(x)$ the probability that x pass through slit 2. If we look at the possibility of the wave-like behaviour of the particle x, then such a (way) particle could be at two locations at the same time and therefore