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Chaotic dynamics of the vibro-impact system under bounded noise perturbation

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ABSTRACT

In this paper, chaotic dynamics of the vibro-impact system under bounded noise excitation is investigated by an extended Melnikov method. Firstly, the Melnikov method in the deterministic vibro-impact system is extended to the stochastic case. Then, a typical stochastic Duffing vibro-impact system is given to application. The analytic conditions for occurrence of chaos are derived by using the random Melnikov process in the mean-square-value sense. In addition, the numerical simulations confirm the validity of analytic results. Also, the influences of interesting system parameters on the chaotic dynamics are discussed.

1. Introduction

Non-smooth models play a prominent role in a range of application areas, including mechanics, biology, and electronic circuits. In recent years, a considerable amount of research into non-smooth dynamical systems (NSDSs), including vibro-impact systems, collision dynamics, chattering dynamics and stick–slip motions, is being actively pursued, see Refs. [1–12]. Current studies show that the dynamics of NSDSs can be very complicated and have many novel dynamical features, such as periodic-adding cascades, non-smooth bifurcations and chattering motion.

As we known, the Melnikov method is a mature approximate tool for investigating the chaotic dynamics in smooth dynamical systems (SDSs), see Refs. [13–17]. However, due to the discontinuousness caused by non-smooth factors, the conventional Melnikov method is not directly appropriate for NSDSs. Comparing to SDSs, the presented results of the Melnikov theory for NSDSs are insufficient. For several kinds of special nonlinear vibro-impact oscillators, a few effective Melnikov methods have been proposed. Du [18] investigated the homoclinic bifurcation and chaos in an inverted impacting pendulum under periodic excitation using the proposed Melnikov method. In the sense of Smale horseshoes, the onset of chaotic motion in a Duffing vibro-impact oscillator with bilateral constrains was discussed by Xu [19].

Unfortunately, all above works are focused to the case of deterministic NSDSs. Many applications of noise have shown a constructive role in statistic physics, biology and engineering. Since Gaussian noise is unbounded and its total power is infinite, Gaussian noise is an inadequate mathematical model. More recently, a vast body of research focused on another important class of non-Gaussian stochastic processes: bounded noises. Therefore, bounded noise is a reasonable model to describe the bounded random excitation. The influences of noise perturbation on chaotic dynamics is important. In the case of noise perturbation, especially for bounded noise, few Melnikov methods are well developed. In the present paper, the Melnikov method is proposed for NSDSs under the bounded noise perturbation.

The paper is organized as follows. In Section 2, the random Melnikov process is discussed in a general vibro-impact oscillator under bounded noise excitation. In Section 3, the Melnikov method described in Section 2 is applied to study the chaotic dynamics for a Duffing

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vibro-impact system. In Section 4, the analytic results are verified numerically, and the influence of interesting parameters on chaotic dynamics is investigated. At last, the conclusions are given in Section 5.

2. Random Melnikov process in a general vibro-impact oscillator

Consider a single-degree-of-freedom vibro-impact oscillator with light damping and noise perturbation. The Hamiltonian system between the rigid walls is expressed as

$$\begin{cases} \dot{Q} = \frac{\partial H}{\partial P}, \\ \dot{P} = -\frac{\partial H}{\partial Q} + \varepsilon\left[-c(Q,P)\frac{\partial H}{\partial P} + b(Q,P,t) + a(Q,P)\xi(t)\right] \end{cases}, h(Q) < 0, \tag{1}$$

and the instantaneous impact law is dominated by the following discrete mapping

$$\begin{pmatrix} Q \\ P \end{pmatrix}_+ = I \cdot \begin{pmatrix} Q \\ P \end{pmatrix}_-, I = \begin{pmatrix} 1 & 0 \\ 0 & -(1-\varepsilon r_0) \end{pmatrix}, \quad h(Q) = 0, \tag{2}$$

where Q and P are generalized displacement and velocity, respectively. ε is the small scale parameter. $c(Q, P)$ denotes the coefficient of damping. $b(Q, P, t)$ represents period excitation. $a(Q, P)$ depicts the amplitude of noise perturbation. $h(Q)$ is the constrain function. The impact mapping I depicts the jump of the state of system. The parameter r_0 describes the loss of energy caused by the impact and satisfies $0 \leqslant r_0 < 1$. The subscripts "−" and "+" in the law (2) denote the instants just before and after impacts, respectively. $\xi(t)$ is a bounded noise, and satisfies

$$\xi(t) = \sin(\omega_2 t + \sigma W(t) + \gamma), \tag{3}$$

where ω_2 is the excitation frequency, and $W(t)$ is a standard Wiener process, γ is a random variable which is uniformly distributed in $[0, 2\pi)$, σ is the noise intensity. The bounded noise $\xi(t)$ is a stationary stochastic process with zero mean value. Its density function is $p(\xi) = \frac{1}{\pi\sqrt{1-\xi^2}}$ and its spectral density function as

$$S_\xi(\omega) = \frac{1}{2\pi}\left[\frac{\sigma^2}{4(\omega-\omega_2)^2+\sigma^4} + \frac{\sigma^2}{4(\omega+\omega_2)^2+\sigma^4}\right]. \tag{4}$$

Obviously, its simple functions are continuous and bounded. Moreover, the bounded noise $\xi(t)$ is a narrow-band process when σ is small and it approaches the white noise when $\sigma \to \infty$.

When $\varepsilon = 0$ in Eqs. (2) and (3), there is a hyperbolic fixed point connected to itself by homoclinic or heteroclinic orbits $(q_0(t), p_0(t))^T$. The random Melnikov process of Eqs. (1) and (2) can be derived by the following formula given in Refs. [19–20]

$$M(t_0) = -A_1 + A_2 - A_3 + A_4, \tag{5}$$

where

$$A_1 = \int_{-\infty}^{+\infty} c(Q,P)\left(\frac{\partial H}{\partial P}\right)^2\bigg|_{Q=q_0(t),\ P=p_0(t)} dt, \tag{6}$$

$$A_2 = \int_{-\infty}^{+\infty} b(Q,P,t+t_0)\frac{\partial H}{\partial P}\bigg|_{Q=q_0(t),\ P=p_0(t)} dt, \tag{7}$$

$$A_3 = 2r_0\int_0^\Delta \left(-\frac{\partial H}{\partial Q}\right)\bigg|_{Q=q_0(t),\ P=p_0(t)} dQ, \tag{8}$$

$$A_4 = \int_{-\infty}^{\infty} a(Q,P)\xi(t+t_0)\frac{\partial H}{\partial P}\bigg|_{Q=q_0(t),\ P=p_0(t)} dt. \tag{9}$$

Δ is the positive root of the equation $h(Q) = 0$. Note that the term A_3 represents the element of impact.

Let $M_d(t_0) = -A_1 + A_2 - A_3$, $M_r(t_0) = A_4$, formula (5) can be rewritten as

$$M(t_0) = M_d(t_0) + M_r(t_0), \tag{10}$$

where $M_d(t_0)$ represents the mean of the random Melnikov process, i.e. $E[M(t_0)] = M_d(t_0)$. $M_r(t_0)$ is the random component with zero mean due to the bounded noise.

The integral $M_r(t_0)$ can be rewritten in the form of convolution

$$M_r(t_0) = l(t) * \xi(t), \tag{11}$$

where $l(t) = a(Q,P)\frac{\partial H}{\partial P}\big|_{Q=q_0(t),\ P=p_0(t)}$ is the impulse response function, while $\xi(t)$ is an input of the system. Thus, the variance of $M_r(t_0)$ can be derived by

$$\sigma_{M_r}^2 = \int_{-\infty}^{+\infty} |H(\omega)|^2 S_\xi(\omega)d\omega, \tag{12}$$

where $S_\xi(\omega)$ has been given by Eq. (4). $H(\omega)$ is the frequency response function of $l(t)$, which is the Fourier transform of the impulse response function, i.e.

$$H(\omega) = \int_{-\infty}^{+\infty} \frac{\partial H}{\partial P} b(Q,P)e^{-j\omega t}dt, \tag{13}$$

see Ref. [17] in detail.

Note that the standard deviation of $M(t_0)$ is σ_{M_r}, which means $M(t_0)$ varies between $M_d - \sigma_{M_r}$ and $M_d + \sigma_{M_r}$. From the view of energy, the random Melnikov process (5) has a simple zero point in mean-square sense when

$$(A_1 + A_3)^2 \le (A_2 + A_4)^2. \tag{14}$$

3. Melnikov analysis in a Duffing vibro-impact oscillator

Consider a typical Duffing vibro-impact oscillator with bilateral constrains, the equation of motion is

$$\begin{cases} \dot{Q} = P, \\ \dot{P} = -Q + \alpha Q^3 + \varepsilon[-\beta P + f\sin(\omega_1 t) + \mu\xi(t)] \end{cases}, |Q| < \Delta, \tag{15}$$

and the impact law is

$$\dot{P}_+ = -r\dot{P}_-, \quad |Q| = \Delta, \tag{16}$$

where α and Δ are positive constants, the constraint function is defined as $h(Q) = |Q| - \Delta$, $\dot{P}_+$ and $\dot{P}_-$ are velocities just before and after impacts, which means $\dot{P}_\pm = \dot{P}(t_* \pm 0)$ for the instants of impact t_*. When the restitute coefficient r satisfies $r = 1 - \varepsilon r_0$, Eq. (16) is of the same form as Eq. (2).