Chaos, Solitons & Fractals 73 (2015) 17–28

Contents lists available at ScienceDirect

Chaos, Solitons & Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

De-synchronization and chaos in two inductively coupled Van der Pol auto-generators

R.Y. Beregov, A.V. Melkikh *

Ural Federal University, Mira str. 19, Yekaterinburg 620002, Russia

ARTICLE INFO

Article history:
Received 15 October 2014
Accepted 22 December 2014
Available online 20 January 2015

ABSTRACT

In this article, we consider a system of autonomous inductively coupled Van der Pol generators. For two coupled generators, we establish the presence of metastable chaos, a strange non-chaotic attractor, and several stable limiting cycles. Areas of parametric dependence of different modes of synchronization are obtained.

1. Introduction

Understanding the mechanisms of the onset of chaos and collective effects in complex systems is an urgent task. As is known, the mutual influence of generators leads to the phenomenon of self-excited oscillation synchronization [1–5], wherein the oscillation frequencies or phases of different generators are close to each other. However, it is also possible for such a connection of auto-generators to enter chaotic modes and modes of de-synchronization of auto-oscillations. The latter phenomenon is much less studied in theoretical terms. Meanwhile, the de-synchronization of generators can lead to quite complex system behavior, as the topology of connections and their signs generate a wide variety of possible structures. In particular, the appearance and properties of chaos should essentially depend on the topology and the types of connections between generators. In this paper, we consider two interacting Van der Pol generators with inductive coupling. This connection may have both positive and negative signs, leading to the synchronization or de-synchronization of auto-oscillations.

2. The system of equations for the two inductively coupled Van der Pol auto-generators

The Van der Pol generator is one of the first studies generators. The Van der Pol equations, despite their long history, widely used now in various fields of science (see, for example, [6–17]). Coupled Van der Pol generators were also considered. The synchronization [4] and emergence of chaos [18] in such systems were studied. Coupled generators are interesting in that they exhibit complex chaotic behavior of such a system. This allows to study the most general properties of chaos. On the other hand, chaotic generators have many technical applications, such as information hiding, etc.

However, because the properties of chaotic behavior [19] should essentially depend on the mechanism and on the coupling strength, it is necessary to consider each mechanism separately. One such mechanism may be inductive coupling between the Van der Pol auto-generators; however, the appearance and parameters of the chaotic behavior in this system remain unexplored. Inductive coupling exists in the generator itself, so it is natural to consider the inductive coupling between the different generators.

Consider the classical equation of a Van der Pol oscillator in dimensionless form:

$$\frac{dx(t)^2}{dt^2} - (\mu - x(t)^2)\frac{dx(t)}{dt} + x(t) = 0, \quad (1)$$

* Corresponding author.
E-mail address: melkikh2008@rambler.ru (A.V. Melkikh).

http://dx.doi.org/10.1016/j.chaos.2014.12.017
0960-0779/

Table 1
Parameters of the system of Eq. (2).

μ_1	μ_2	M_{12}
1	1	0.5

where $\mu = \frac{Mg_0 - RC}{\sqrt{LC}}$, C – is the capacitance of the circuit, L – is the coefficient of self-induction, R – is the resistance, M – is the coefficient of mutual induction of the RCL circuit with a non-linear amplifier circuit, and g_0 – is the non-linear characteristic of the amplifier,

$$i(U(t)) = g_0 U(t) - g_2 U(t)^3.$$

There are many variants for the connection between the auto-generators [1,20,4,5]. Consider the case of inductive coupling of two identical circuits. In this case, the system of equations can be written as

$$\begin{aligned}\frac{dx_1(t)^2}{dt^2} - (\mu_1 - x_1(t)^2)\frac{dx_1(t)}{dt} + x_1(t) &= -M_{12}\frac{dx_2(t)^2}{dt^2},\\ \frac{dx_2(t)^2}{dt^2} - (\mu_2 - x_2(t)^2)\frac{dx_2(t)}{dt} + x_2(t) &= -M_{12}\frac{dx_1(t)^2}{dt^2},\end{aligned} \tag{2}$$

wherein M_{12} is the coefficient of mutual induction, which can be either positive or negative depending on the mutual arrangement of the coils.

3. Dynamic modes of the system

Non-hyperbolic systems may have complex basins of attraction, where part of the attractors can be generally impossible to trace because of the limited accuracy of numerical calculations. We illustrate this phenomenon with several examples.

Let us choose, for example, to follow the oscillator parameters (Table 1).

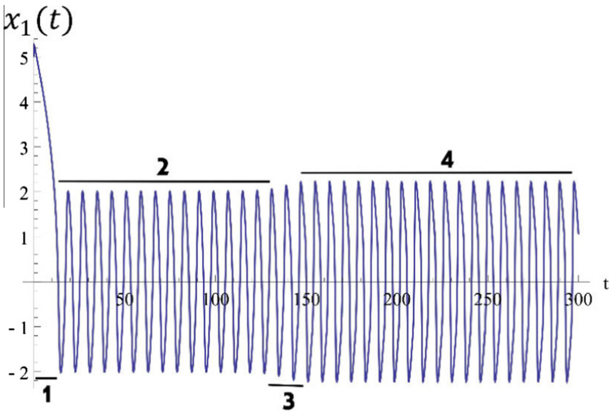

Fig. 1. The diagram shows several stages of evolution of the system, namely: phase 1 corresponds to the transition process ($t \in (0, 10)$), phase 2 corresponds to a metastable regime ($t \in (10, 130)$), step 3 again corresponds to the transition process ($t \in (130, 150)$), and step 4 corresponds to staying on the attractor ($t \in (150, \infty)$). The graph is plotted for the initial conditions $x_1(0) = 5$, $x_1'(0) = 5$, $x_2(0) = 5$, $x_2'(0) = 5$.

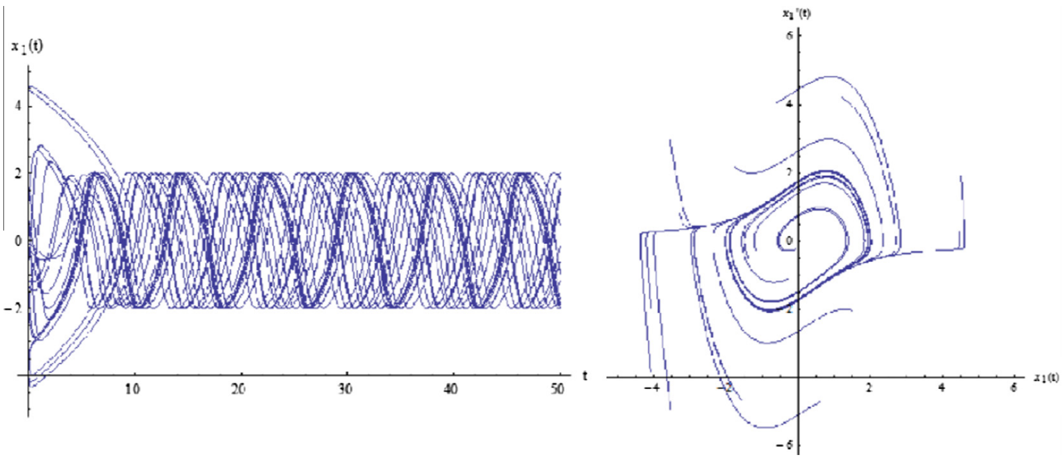

Fig. 2. Plots the evolution of the set of different initial conditions. It is easy to notice that the system always falls on the same trajectory in the phase space, and the phase trajectories are constructed using the coordinates of the first generator (step 2 in Fig. 1).