Chaos, Solitons & Fractals 73 (2015) 64–70

Contents lists available at ScienceDirect

Chaos, Solitons & Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Distributions of city sizes in Mexico during the 20th century

Enrique Pérez-Campuzano [a], Lev Guzmán-Vargas [b,*], F. Angulo-Brown [c]

[a] Instituto de Geografía, Circuito Exterior, Col. Universidad Nacional Autónoma de México, UNAM, México D.F. 04510, Mexico
[b] Unidad Profesional interdisciplinaria en Ingeniería y Tecnologías Avanzadas, Instituto Politécnico Nacional, Av. IPN No. 2580, México D.F. 07340, Mexico
[c] Departamento de Física, Escuela Superior de Física y Matemáticas, Instituto Politécnico Nacional, Edif. No. 9 U.P. Zacatenco, México D.F. 07738, Mexico

ARTICLE INFO

Article history:
Received 19 November 2014
Accepted 19 December 2014
Available online 21 January 2015

ABSTRACT

We present a study of the distribution of cities in Mexico along the 20th century, based on information collected in censuses every ten years. The size-rank and survival cumulative distributions are constructed to evaluate the presence of scaling, its deviation from the Zipf's law and their evolution along the period of observation. We find that the size of cities S, approximately follow a power-law with the rank $r, S(r) \sim r^{-\alpha}$, where the exponents take values between $\alpha \approx 0.7$ to $\alpha \approx 1.1$ for years 1900 and 2000, respectively. The local fluctuations in the scaling behavior are evaluated by means of a local exponent, and the deviation of the size predicted by the Zipf's law $(\alpha = 1)$ and the real size of each city is analyzed. Our calculations show that local exponents follow transitions between values above and below of the Zipfian regime and the deviations are more remarkable at the beginning and at the end of the 20th century. Besides, the cumulative distributions confirm the presence of scaling for the same records with a reasonable agreement with the scaling exponents observed in the size-rank distributions. Moreover, we examine the role of a recent introduced property named coherence. Finally, we explain our findings in terms of the socio-demographic evolution of Mexico along the 20th century.

1. Introduction

Since Pareto [1] in the 19th century up to the present day, many complex collective phenomena have been statistically described by scaling laws. This approach has been used in a variety of research fields ranging from social to natural sciences [2]. For example, from economics [3–5], linguistics [6,7] and sociology [8] to physics [9–11] and biology [12]. The phenomena described by scaling laws have no characteristic scales and usually are described by power laws of the fractal type. Scaling relationships were first found on empirical grounds. A clear case of this is that of the empirical laws of seismology, such as the so-called Omori law for aftershocks temporal distribution [13] or the Gutenberg–Richter law for the magnitude distribution of seisms [14]. Several decades after these laws were established empirically, Bak et al. [15,16] proposed a more fundamental explanation for them based on the concept of self-organized critical systems. The same happened with the scaling laws of the Zipf type, followed by the distribution of cities by size [17], which were first empirically established and until recent times more detailed mechanisms for their explanation have been proposed [18,19]. On the other hand, as it is well known, only the self-similar fractals, as the Koch's curve for instance, have scaling laws over an arbitrary number of scales. However, the scaling properties in real world objects and phenomena are incomplete in the sense that the corresponding power laws typically have one or more crossovers in their scaling exponents [20]. For example, the Gutenberg–Richter law for earthquakes extends along many decades from millimeters up to thousands of kilometers, however, it has a crossover

* Corresponding author.
E-mail addresses: eperez@igg.unam.mx (E. Pérez-Campuzano), lguzmanv@ipn.mx (L. Guzmán-Vargas), angulo@esfm.ipn.mx (F. Angulo-Brown).

http://dx.doi.org/10.1016/j.chaos.2014.12.015

around of $M = 7.5$, which has to do with a break in self-similarity, from small to large earthquakes occurring at a point where the dimensions of the event equals the down-dip width of the seismological layer (see Fig. 1 of Ref. [21]). Small earthquakes grow in both length and width; their rupture dimensions have no bounds. On the other hand, large earthquakes have no bounds in rupture length, but their down-dip width is limited by thickness of the region capable of generating earthquakes [21].

Another case where a crossover plays a very important role is that corresponding to empirical data of distribution of the money and wealth in many countries as in UK for example [22,23]. When a log–log plot of cumulative probability distribution against total of capital (wealth) is made, clearly two well defined regions are found, one following a Pareto-type behavior for rich people and other following an experimental distribution of the Boltzmann-type for the lower part of the distribution for the great majority (about 90%) of the population. Some well-founded models explain these different behaviors [22,24–26]. For the case of the city size distribution, deviations from Zipf's law and crossovers have been reported [27–30]. As in the case of the money and wealth distributions, crossovers between big ($S > 10^5$ inhabitants) and small scales, and between developed and developing countries have been identified. For the case of Japan for instance, Sasaki et al. reported that the rank-size distributions of towns and villages can be well approximated by log normal distributions while for cities a power-law distribution is observed. On the other hand, Malacarne et al. [31] also have reported that for the cases of Brazil and USA a notorious deviation from an asymptotic power-law of the Zipf type is found when cities of all sizes are considered. Until the present day, there is no consensus about the best fit of city size distributions. For example, Bee et al. [32] question the claim that largest US cities are Pareto distributed and based on multiple tests on real data they assert that the distribution is lognormal, and largely depends on sample sizes. On the other hand, Giesen et al. [33] by using untruncated settlement size data from eight countries show that the "double Pareto lognormal" distribution provides a better fit to actual city sizes than the simple lognormal distribution. More recently, Cristelli et al. [27] have reported a very important property called "coherence" of the sample of objects to be studied, which drives the resulting power-law towards a perfect Zipf's law ($\alpha = 1$) or towards marked deviations from it. Another important feature of city size distributions (CSD), is the time evolution of the power-law exponents depending of many social, economic and political phenomena [27,31]. The CSD has been studied for many countries as USA ([34]), Spain [35], France [28], Japan [28], India [36], China [36,37], Brazil [38] among others. The study of particular cases is important because they help to enrich the phenomenology of the CSD problem. In the present article we study the case of Mexico by analyzing each decade censuses from 1900 up to 2000. The main goal of this work is to evaluate presence and departures of the scaling behavior in CSD, and deviations with respect to the Zipf's regime. Our results clearly reflect some of the main social and economic transitions of Mexico along the 20th century under a macroscopic overview through the evolution of the CSD scaling exponents. Our paper is organized as follows. In Section 2, we describe the main aspects of the data of cities under study and some sociodemographic aspects which are important for the discussion of the results. In Section 3, the results are presented together with a discussion, and finally in Section 4 some concluding remarks are given.

2. Data and sociodemographic aspects

One of the big issues in the construction of the relationship between population and rank of cities is the definition of a city [36]. Some authors use a spatial definition for cities, particularly based on the amount of building area. Other studies [39] use an administrative definition, i.e. government of specific states or cities. In the case of this study, we combine both perspectives. In first place, metropolitan zones are defined as municipalities agglomerations; that is, data corresponding to the total of the population of the municipalities and they were taken from different sources [38,40,41]. We follow the common practice in Mexico to assume the size of 15 000 inhabitants as the lower threshold to define a city [38]. Although there are some criticisms to this threshold, it remains as the most extended definition used in Mexico (our dataset can be freely accessed on the website *http://www.cslupiita.com.mx/citiesmx/*).

Along the 20th century, as many countries in the world, Mexico passed through important transformations, including its transition from a rural to an urban country [42]. As can be seen in Fig. 1, the annual growth rate (AGR) of the urban population was higher than the AGR of total population for the considered period. In the 1950's the largest difference between both AGR's is observed due to the industrialization policy, the promotion of the urbanization, and rural to urban migration. Thanks to the population policies since the 1970's, it seems that nowadays both AGR's are close each other. In 1900, just the 10.6% of the population lived in one of the 33 Mexican cities. By 2000, 364 cities formed the Mexican urban system with more than 60 million inhabitants (63% of the total).

Some authors such as Unikel [38,43], Garza [40] and Aguilar and Graizbord [42] have pointed out the importance of dividing the recent urban history of Mexico in three periods: I. 1900–1939, II. 1940–1979 and III. 1980–2000 (see Table 1). In 1900, Mexico was a rural country. The urban population (10.6%) was distributed in 33 cities and six of them concentrated the one half of the total. During the first period the percentage of urban population grew around 7% meanwhile the total of cities reached 43 (i.e. 30%).

In the second period (1940–1979), the process of urbanization took place in a faster rate due to a state policy of industrialization and urbanization. By the beginning of this period, around 20% of the total of population lived in 55 cities, meanwhile by the end the urban population reached 46% and the total of cities was 167. In other words, urban population doubled meanwhile the total of cities tripled.

The last period (named the neoliberal one), witnessed notable changes. In the first place the economic openness