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Review

Percolation of interdependent network of networks

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ABSTRACT

Complex networks appear in almost every aspect of science and technology. Previous work in network theory has focused primarily on analyzing single networks that do not interact with other networks, despite the fact that many real-world networks interact with and depend on each other. Very recently an analytical framework for studying the percolation properties of interacting networks has been introduced. Here we review the analytical framework and the results for percolation laws for a *Network Of Networks* (NONs) formed by n interdependent random networks. The percolation properties of a network of networks differ greatly from those of single isolated networks. In particular, because the constituent networks of a NON are connected by node dependencies, a NON is subject to cascading failure. When there is strong interdependent coupling between networks, the percolation transition is discontinuous (first-order) phase transition, unlike the well-known continuous second-order transition in single isolated networks. Moreover, although networks with broader degree distributions, e.g., scale-free networks, are more robust when analyzed as single networks, they become more vulnerable in a NON. We also review the effect of space embedding on network vulnerability. It is shown that for spatially embedded networks any finite fraction of dependency nodes will lead to abrupt transition.

1. Introduction

The interdisciplinary field of network science has attracted great attention in recent years [1–27]. This has taken place because an enormous amount of data regarding social, economic, engineering, and biological systems has become available over the past two decades as a result of the information and communication revolution brought about by the rapid increase in computing power. The investigation and growing understanding of this extraordinary amount of data will enable us to make the infrastructures we use in everyday life more efficient and more robust. The original model of networks, random graph theory, developed in the 1960s by Erdős and Rényi (ER), is based on the assumption that every pair of nodes is randomly connected with the same probability (leading to a Poisson degree distribution). In parallel, lattice networks in which each node has the same number of links have been used in physics to model physical systems. While graph theory was a well-established tool in the mathematics and computer science literature, it could not adequately describe modern, real-world networks. Indeed, the pioneering observation by Barabási in 1999 [2], that many real networks do not follow the ER model but that organizational principles naturally arise in most systems, led to an overwhelming accumulation of supporting data, new models,

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and novel computational and analytical results, and led to the emergence of a new and very active multidisciplinary field: network science.

Significant advances in understanding the structure and function of networks, and mathematical models of networks have been achieved in the past few years. These are now widely used to describe a broad range of complex systems, from techno-social systems to interactions amongst proteins. A large number of new measures and methods have been developed to characterize network properties, including measures of node clustering, node centrality, network modularity, correlation between degrees of neighboring nodes, measures of node importance, and methods for the identification and extraction of community structures. These measures demonstrated that many real networks, and in particular biological networks, contain network motifs—small specific subnetworks—that occur repeatedly and provide information about functionality [9]. Dynamical processes, such as flow and electrical transport in heterogeneous networks, were shown to be significantly more efficient compared to ER networks [28,29].

Complex networks are usually non-homogeneous structures that exhibit a power-law form in their degree (number of links per node) distribution. These systems are called scale-free networks [30]. Some examples of real-world scale-free networks include the Internet [3], the WWW [4], social networks representing the relations between individuals, infrastructure networks such as airlines [31,32], networks in biology, in particular networks of protein–protein interactions [33], gene regulation, and biochemical pathways, and networks in physics, such as polymer networks or the potential energy landscape network. The discovery of scale-free networks has led to a re-evaluation of the basic properties of networks, such as their robustness, which exhibit a character that differs drastically from that of ER networks. For example, while homogeneous ER networks are vulnerable to random failures, heterogeneous scale-free networks are extremely robust [4,5]. Much of our current knowledge of networks is based on ideas borrowed from statistical physics, e.g., percolation theory, fractal analysis, and scaling analysis. An important property of these infrastructures is their stability, and it is thus important that we understand and quantify their robustness in terms of node and link functionality. Percolation theory was introduced to study network stability and to predict the critical percolation threshold [5]. The robustness of a network is usually (i) characterized by the value of the critical threshold analyzed using percolation theory [34] or (ii) defined as the integrated size of the largest connected cluster during the entire attack process [35]. The percolation approach was also extremely useful in addressing other scenarios, such as efficient attacks or immunization [6,8,15,36,37], for obtaining optimal path [38] as well as for designing robust networks [35]. Network concepts were also useful in the analysis and understanding of the spread of epidemics [39,40], and the organizational laws of social interactions, such as friendships [41,42] or scientific collaborations [43]. Moreira et al. investigated topologically-biased failure in scale-free networks and controlled the robustness or fragility by fine-tuning the topological bias during the failure process [44].

Because current methods deal almost exclusively with individual networks treated as isolated systems, many challenges remain [45]. In most real-world systems an individual network is one component within a much larger complex multi-level network (a specific type of a network of networks). As technology has advanced, coupling between networks has become increasingly strong. Node failures in one network will cause the failure of dependent nodes in other networks, and vice versa [46]. This recursive process can lead to a cascade of failures throughout the network of networks system. The study of individual particles has enabled physicists to understand the properties of a gas, but in order to understand and describe a liquid or a solid the interactions between the particles also need to be understood. So also in network theory, the study of isolated single networks brings extremely limited results—real-world noninteracting systems are extremely rare in both classical physics and complex systems. Most real-world network systems continuously interact with other networks, especially since modern technology has accelerated network interdependency.

To adequately model most real-world systems, understanding the interdependence of networks and the effect of this interdependence on the structural and functional behavior of the coupled system is crucial. Introducing coupling between networks is analogous to the introduction of interactions between particles in statistical physics, which allowed physicists to understand the cooperative behavior of such rich phenomena as phase transitions. Surprisingly, preliminary results on mathematical models [46,47] show that analyzing complex systems as a network of coupled networks may alter the basic assumptions that network theory has relied on for single networks. Here we will review the main features of the theoretical framework of Network of Networks, NON [48,49], and present some real world applications.

2. Overview

In order to model interdependent networks, we consider two networks, A and B, in which the functionality of a node in network A is dependent upon the functionality of one or more nodes in network B (see Fig. 1, and vice versa: the functionality of a node in network B is dependent upon the functionality of one or more nodes in network A. The networks can be interconnected in several ways. In the most general case we specify a number of links that arbitrarily connect pairs of nodes across networks A and B. The direction of a link specifies the dependency of the nodes it connects, i.e., link $A_i \rightarrow B_j$ provides a critical resource from node A_i to node B_j. If node A_i stops functioning due to attack or failure, node B_j stops functioning as well but not vice versa. Analogously, link $B_i \rightarrow A_j$ provides a critical resource from node B_i to node A_j.

To study the robustness of interdependent networks systems, we begin by removing a fraction $1 - p$ of network A nodes and all the A-edges connected to these nodes. As an outcome, all the nodes in network B that are dependent