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Twisted stacked central configurations for the spatial seven-body problem

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ABSTRACT

In this paper, we show the existence of the twisted stacked central configurations for 7-body problem. More precisely, the position vectors x_1, x_2, x_3 and x_4 are at the vertices of a regular tetrahedron Σ; the position vectors x_5, x_6 and x_7 are at the vertices of an equilateral triangle Π; the triangle (x_1, x_2, x_3) and the triangle (x_5, x_6, x_7) have twisted angle $\frac{\pi}{3}$.

1. Introduction and main results

The classical n-body problem concerns with the motion of n mass points moving in space according to Newton's second law and the gravitational law:

$$m_i\ddot{x}_i = \sum_{k\neq i} \frac{m_k m_i (x_k - x_i)}{|x_k - x_i|^3}, \quad i = 1, 2, \ldots, n. \tag{1.1}$$

Here $x_i \in \mathbb{R}^3$ is the position of mass $m_i > 0$. Alternatively, system (1.1) can be rewritten as

$$m_i\ddot{x}_i = \frac{\partial U(x)}{\partial x_i}, \quad i = 1, 2, \ldots, n \tag{1.2}$$

where

$$U(x) = U(x_1, x_2, \ldots, x_n) = \sum_{1\leq k<j\leq n} \frac{m_k m_j}{|x_k - x_j|} \tag{1.3}$$

is the Newtonian potential of system (1.1). The position vector $x = (x_1, x_2, \ldots, x_n) \in (\mathbb{R}^3)^n$ is often referred to the configuration of the system; the vectors $x_i (i = 1, 2, \ldots, n)$ are vertices of the configuration x.

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Let $M = m_1 + \cdots + m_n$ be the total mass and

$$c = \frac{1}{M}(m_1 x_1 + \cdots + m_n x_n)$$

be the center of mass of the configuration x. Because the potential is singular when two particles have the same position, it is natural to assume that the configuration avoids the set

$$\triangle = \{x \in (\mathbb{R}^3)^n : x_i = x_j \text{ for some } i \neq j\}.$$

A configuration $x = (x_1, \ldots, x_n) \in (\mathbb{R}^3)^n \setminus \triangle$ is called a *central configuration* if there exists some positive constant λ, called the multiplier, such that

$$-\lambda(x_i - c) = \sum_{j=1, j\neq i}^{n} \frac{m_j(x_j - x_i)}{|x_j - x_i|^3}, \quad i = 1, 2, \ldots, n. \tag{1.4}$$

It is easy to see that a central configuration remains a central configuration after a rotation in $\mathbb{R}^3$ and a scalar multiplication. More precisely, let $A \in SO(3)$ and $a > 0$, if $x = (x_1, \ldots, x_n)$ is a central configuration, so are $Ax = (Ax_1, Ax_2, \ldots, Ax_n)$ and $ax = (ax_1, ax_2, \ldots, ax_n)$.

Two central configurations are said to be equivalent if one can be transformed to another by a scalar multiplication and a rotation. In this paper, when we say a central configuration, we mean a class of central configurations as defined by the above equivalence relation.

The study of central configuration goes back to Euler and Lagrange. For $n = 3$, it is a classical result there are three collinear, called Euler, central configurations and one equilateral triangular, called Lagrange, central configurations. For $n = 4$, Moulton [1] proved that there is exactly one collinear central configuration for each arrangement of the mass points on the line.

There are several reasons why central configurations are of special importance in the study of the n-body problem; see [2–4] for details.

A *stacked central configuration* is a central configuration in which a proper subset of the n bodies is already on a central configuration. This class of central configuration of 5-body problem was introduced by Hampton in [5]. The work of [5] was complemented by Llibre in [6,7].

Zhang and Zhou [8] showed the existence of double pyramidal central configurations of $N + 2$-body problem. Hampton and Santoprete [9] provided some examples of stacked central configurations for the spatial 7-body problem where the bodies are arranged as concentric three and two dimensional simplex.

Mello and Fernandes [10] provided new examples of stacked central configuration for spatial 7-body problem where the four bodies are at the vertices of a regular tetrahedron and the other three bodies are located at the vertices of an equilateral triangle in the exterior of regular tetrahedron. In this paper, we show the existence of the twisted stacked central configurations of the 7-body problem. The spatial central configuration considered here satisfies (see Fig. 1): the position vectors x_1, x_2, x_3 and x_4 are at the vertices of a regular tetrahedron Σ; the position vectors x_5, x_6 and x_7 are at the vertices of an equilateral triangle Π; the triangle (x_1, x_2, x_3) and the triangle (x_5, x_6, x_7) have twisted angle $\frac{\pi}{3}$.

Without loss of generality, we can assume that

$$\begin{aligned} &x_1 = (1, 0, 0), \quad x_2 = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right), \quad x_3 = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}, 0\right), \quad x_4 = (0, 0, \sqrt{2}), \\ &x_5 = \left(\frac{1}{2}x, \frac{\sqrt{3}}{2}x, z\right), \quad x_6 = (-x, 0, z), \quad x_7 = \left(\frac{1}{2}x, -\frac{\sqrt{3}}{2}x, z\right), \end{aligned} \tag{1.5}$$

where $x > 0$ is the radius of the circle that contains the equilateral triangle Π and $z \in \mathbb{R}$ is the signed distance between the plane that contains x_1, x_2 and x_3 and the plane that contains Π.

The main results of this paper are the following.

Theorem 1.1. *According to Fig. 1, in order that the seven mass points are in a central configuration, the following statements are necessary:*

1. The masses m_1, m_2 and m_3 must be equal.
2. The masses m_5, m_6 and m_7 must be equal.

Theorem 1.2. *There exist points $P_0(x_0, y_0) \in T^{-1}(0) \cap D_1$ such that the seven bodies take the coordinates*

$$\begin{aligned} &x_1 = (1, 0, 0), \quad x_2 = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right), \quad x_3 = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}, 0\right), \quad x_4 = (0, 0, \sqrt{2}), \\ &x_5 = \left(\frac{1}{2}x_0, \frac{\sqrt{3}}{2}x_0, z_0\right), \quad x_6 = (-x_0, 0, z_0), \quad x_7 = \left(\frac{1}{2}x_0, -\frac{\sqrt{3}}{2}x_0, z_0\right). \end{aligned}$$

Then there are positive solutions of m_1, m_4, m_5 such that these bodies form a spatial central configuration according to Fig. 1.