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Chaos suppression on a class of uncertain nonlinear chaotic systems using an optimal H_∞ adaptive PID controller

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ABSTRACT

This paper introduces an optimal H_∞ adaptive PID (OHAPID) control scheme for a class of nonlinear chaotic system in the presence system uncertainties and external disturbances. Based on Lyapunov stability theory, it is shown that the proposed control scheme can guarantee the stability robustness of closed-loop system with H_∞ tracking performance. In the core of proposed controller, to achieve an optimal performance of OHAPID, the Particle Swarm Optimization (PSO) algorithm is utilized. To show the feasibility of proposed OHAPID controller, it is applied on the chaotic gyro system. Simulation results demonstrate that it has highly effective in providing an optimal performance.

1. Introduction

Chaotic system is a highly nonlinear system and its prominent characteristic is aperiodic dynamics with sensitive dependence on initial conditions. Aperiodic is simply the behavior that never repeats whereas sensitivity means that a small change in the initial state will lead to progressively larger changes in later system states. This sensitivity makes the motion unpredictable and irregular in the long run. The study of chaotic systems is motivated by the fact that many interesting natural and manmade phenomena such as earthquakes and laser systems are with chaotic nature. Over the last two decades, many approaches have been widely applied to control and synchronize chaotic systems such as sliding mode control [1–4], adaptive control [5], state feedback control [6] and so on.

Although several control methods have been proposed for the control of nonlinear systems, proportional-integral-derivative (PID) controllers are still the most common controllers in the process control, due to their simple structure, easy realization, and high reliability. Nevertheless, the performance of PID controllers depends on their parameters absolutely. For an ideal control performance by the PID controller, an appropriate PID parameters tuning is necessary [7]. To tune these parameters, one can use the well-known tuning formula proposed by Ziegler and Nichols (ZN) [8]. But, traditional tuning methods such as ZN are designed based on the mathematical models of systems being controlled. Recently, H_∞ tracking control theory has been developed in robust stabilization and disturbance attenuation [9–12].

Motivated by the aforementioned researches, this paper proposes an Adaptive PID (HAPID) controller with H_∞ tracking performance for control of chaotic systems in the presence of system uncertainties and external disturbance. By the Lyapunov stability theory, it is shown that the proposed controller guarantees the H_∞ tracking performance to attenuate the lumped uncertainties caused by the unmodeled dynamics and external disturbance. Simulation results applied on the gyro chaotic system present the validity of HAPID controller. But, when applying the aforementioned adaptive control manner into the system, an error dynamic of closed-loop system is obtained. Accordingly, we face with choosing an arbitrary set of pole position for the error dynamic. It is unclear how the locations of closed-loop poles relate to the controller performance. It seems that control engineer attempts several different pole locations by trial and error until a desirable response is obtained. To overcome this shortage, this paper introduces an Optimal HAPID (OHAPID) by utilizing a heuristic optimization algorithm namely Particle Swarm Optimization (PSO) to determine the appropriate poles

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location. The simplicity of the proposed OHAPID controller provides a novel approach to control a variety of nonlinear chaotic systems.

The rest of paper is organized as follows. The system description and preliminaries are given in Section 2. In Section 3, HAPID controller is designed. Moreover, a theorem to show the control performance of the closed-loop system is demonstrated. Section 4 presents a procedure to design the proposed OHAPID controller. Simulation results are given in Section 5 to show the feasibility of the proposed approach. Section 6 concludes this paper.

2. System description and problem formulation

The problem of chaos control is to force a chaotic system to exhibit a prefixed non-chaotic behavior and to do so robustly respect to parameter uncertainties. In order to investigate the problem in this paper, the following general controlled chaotic system is considered

$$\begin{aligned} x^{(n)} &= f\left(x, \dot{x}, \ldots, x^{(n-1)}\right) + b\left(x, \dot{x}, \ldots, x^{(n-1)}\right)u \\ y &= x \end{aligned} \tag{1}$$

where $u \in \mathrm{R}$ is the control input, $\mathbf{x} = \left[x, \dot{x}, \ldots, x^{(n-1)}\right]^T = [x_1, x_2, \ldots, x_n]^T \in R^n$ is the state vector, and $f(\mathbf{x})$ and $b(\mathbf{x})$ are unknown (uncertain) but bounded nonlinear functions. Taking the consideration of system uncertainties, the system given in Eq. (1) can be written as:

$$\begin{aligned} x^{(n)} &= f(\mathbf{x}) + b(\mathbf{x})u + d \\ y &= x \end{aligned} \tag{2}$$

where d is the external bounded disturbance. So the chaos control problem is that the system output $\mathbf{y}$ can follow the reference signal $\mathbf{y}_d$.

The following assumptions have to be stated first:

Assumption 1. The value of n is known and the existence of the solution for Eq. (2) is satisfied.

Assumption 2. The input gain $b(\mathbf{x})$ is strictly positive. It means that $b(\mathbf{x}) \geqslant b_{\min}(\mathbf{x}) > 0$.

Let define the tracking error vector

$$\mathbf{e} = \mathbf{y} - \mathbf{y}_d \tag{3}$$

where $\mathbf{e} = \left[e, \dot{e}, \ldots, e^{(n-1)}\right]^T \in R^n$, $\mathbf{y} = \left[y, \dot{y}, \ldots, y^{(n-1)}\right]^T \in R^n$, and $\mathbf{y_d} = \left[y_d, \dot{y}_d, \ldots, y_d^{(n-1)}\right]^T \in R^n$. If the functions $f(\mathbf{x})$ and $b(\mathbf{x})$ in Eq. (2) are known, we have

$$\mathbf{e}^{(n)} = -\mathbf{k}^T\mathbf{e} + b(\mathbf{x})(u - u^*) + d \tag{4}$$

then the control law is obtained as

$$u^* = \frac{1}{b(\mathbf{x})}\left[-f(\mathbf{x}) + y_d^{(n)} + \mathbf{k}^T\mathbf{e}\right] \tag{5}$$

where $\mathbf{k} = [k_1, k_2, \ldots, k_n]^T \in R^n$ is chosen such that all the roots of closed-loop system is a Hurwitz polynomial. On the other hand, all the roots of $\mathbf{k}^T\mathbf{e} = 0$ lie in the left hand side of the s-plane. Therefore, the error dynamic of closed-loop system can be written as

$$\dot{\mathbf{e}} = \mathbf{A}\mathbf{e} + \mathbf{B}[b(\mathbf{x})(u - u^*) + d] \tag{6}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ -k_1 & -k_2 & \cdots & \cdots & -k_n \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \tag{7}$$

Remark 1. The control law u^* in Eq. (5) is only true if $b(\mathbf{x}) \neq 0$. This subject has been considered in Assumption 2.

Remark 2. It is clearly obvious that if the nonlinear functions $f(\mathbf{x})$ and $b(\mathbf{x})$ are completely known, the system is free of the external disturbance d and $u = u^*$. In this case, we obtain $\dot{\mathbf{e}} = \mathbf{A}\mathbf{e}$. As a result, the system output converges to the reference input signal asymptotically. It means that $\mathbf{e}(t) \to 0$ as $t \to \infty$. Also mathematically elegant, the above control approach presents a major drawback posed by the requirement of complete knowledge of the system dynamics. In case when only an approximate model of the system is available, the controller given in Eq. (5) cannot be applied for the system. Due to this reason, the control law must be modified. In the next section, it will be described how we can overcome this problem.

3. Design of the adaptive PID controller with H_∞ performance (HAPID)

The PID controller is the standard tool for industrial automation. The flexibility of controller makes it possible to use PID control in many applications. Fig. 1 depicts the general frame work of PID Controller design. The continuous control law of PID controller is

$$u_{PID} = k_p e(t) + k_i \int_0^t e(\tau)\,d\tau + k_d \frac{d}{dt} e(t) \tag{8}$$

where u_{PID} is the PID control force, parameters k_p, k_i and k_d are the proportional, integral, and derivative gains of the controller, respectively. From Eq. (8), the PID control law can be written in a matrix form as

$$u_{PID}(\mathbf{e}|\boldsymbol{\theta}) = \boldsymbol{\theta}^T \boldsymbol{v}(e) \tag{9}$$

where

$$\boldsymbol{\theta}^T = [k_p, k_i, k_d] \tag{10}$$

$$\boldsymbol{v}(e) = \left[e(t), \int_0^t e(\tau)\,d\tau, \frac{d}{dt} e(t)\right]^T \tag{11}$$

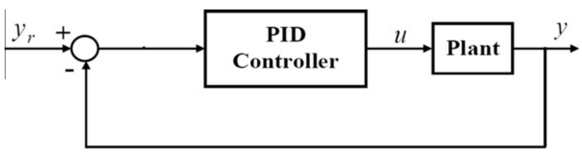

Fig. 1. A general framework of PID controller design.