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Chaos and optimal control of a coupled dynamo with different time horizons

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Abstract

The dynamo system, for acceptable parameter values, exhibits a chaotic behavior. The present paper discusses the problem of chaos, and optimal control of the dynamo system within finite and infinite time horizons. The optimal control inputs that ensure asymptotic stability of this system about its equilibrium states in both cases are obtained as functions of the phase state and time. Extensive numerical studies of both uncontrolled and controlled dynamo system are introduced.

Keywords: Disk dynamos system; Time horizon; Optimal control; Liapunov function; Attractors; Limits cycles

1. Introduction

Recently a considerable amount of research is devoted to study the chaotic behavior of nonlinear dynamical systems. Research effort have investigated the chaos control and chaos synchronization problems in many physical and engineering chaotic systems see for example [1,2,6,7,14,15]. In the present paper our attention is concentrated to give a solution for optimal control and chaos of coupled dynamo system within infinite and finite planning horizon. Many of authors have been discussed the problem of controlling chaos of the coupled dynamo system using different approaches see for example [9–12]. This problem has received a great deal of attention in the literature. Some of these researches directed toward the special case in which the parameters ϵ_1 and ϵ_2 are neglected and torques q_1 and q_2 that applied to the rotors of disk dynamo system are unified and equal one.

The aim of this paper is to study the linear stability analysis and chaotic behavior of the dynamo system and therefore, stabilizes the unstable equilibrium states of this system. Further the studies in this paper complements and extended of our pervious study [9].

The present paper is organized as follows: Following this introduction of Section 1, the problem of coupled dynamo system is formulated in Section 2. The problem of optimal control of the coupled dynamo system within infinite time

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horizon is studied in Section 3. Also this section includes the Lyapunov technique and Hamilton–Jacobi–Bellman method to study the optimal control of equilibrium states of the coupled dynamo system. The control Law is obtained from the conditions that ensure the asymptotic stability of the equilibrium states and minimize the require performance optimality measure. Further, the optimal control of the coupled dynamo system within finite planning horizon is studied in Section 4. Section 5 contains analysis and extensive numerical simulation of the results. Finally, conclusion of the obtained results is presented in Section 6.

2. The mathematical model

This section presents the mathematical model of disk dynamo system and discuss its equilibrium states using linear stability approach. The chaotic behavior of this system will be discussed.

The system of coupled dynamos is a physical nonlinear system which is capable of different chaotic behaviors depending upon both system parameters and initial states [1–3]. The mechanical system of disk dynamos consists of a set of dynamos connected together so that the current generated by any one of them produces the magnetic field for another. Taking for simplicity, the case since there are only two dynamos, we denote the angular velocities of their rotors by ω_1, ω_2 and the currents generated by x_1, x_2, respectively. Then, with appropriate normalization of variables, the dynamical equations can be described by the following set of nonlinear ordinary differential equations [4]:

$$\left.\begin{aligned} \dot{x}_1 &= -\mu_1 x_1 + \omega_1 x_2, \\ \dot{x}_2 &= -\mu_2 x_2 + \omega_2 x_1, \\ \dot{\omega}_1 &= q_1 - \epsilon_1 \omega_1 - x_1 x_2, \\ \dot{\omega}_2 &= q_2 - \epsilon_2 \omega_2 - x_1 x_2, \end{aligned}\right\} \tag{2.1}$$

where q_1 and q_2 are the torques applied to the rotors, and μ_1, μ_2, ϵ_1 and ϵ_2 are positive constants representing dissipative effects of the disk dynamo system.

Next, we discuss the chaos and stability analysis of the disk dynamo system using linear stability approach.

2.1. Chaos and stability analysis

A deep understanding of the behavior of the system (2.1) can be reached by introducing all the system equilibrium states and studying the behavior of this system about these states.

The real equilibrium points of the system (2.1) are

$$E_1 = (\beta_2, \beta_1, \alpha_1, \alpha_2), \quad E_2 = (-\beta_2, -\beta_1, \alpha_1, \alpha_2), \quad E_3 = (0, 0, \gamma_1, \gamma_2), \tag{2.2}$$

where β_i, α_i and γ_i are given by

$$\left.\begin{aligned} &\beta_i = \sqrt{\tfrac{\mu_i(q_i - \epsilon_i \alpha_i)}{\alpha_i}}, \quad \gamma_i = \tfrac{q_i}{\epsilon_i}, \quad i = 1, 2, \\ &\alpha_1 = \sqrt{\tfrac{\mu_1 \mu_2 \epsilon_2}{\epsilon_1}}, \quad \alpha_2 = \sqrt{\tfrac{\mu_1 \mu_2 \epsilon_1}{\epsilon_2}}, \quad q_1 = q_2 = q. \end{aligned}\right\} \tag{2.3}$$

It should be noted that, there are two another complex equilibrium points which are $E_4 = (\beta_2, \beta_1, -\alpha_1, -\alpha_2)$, and $E_5 = -(\beta_2, \beta_1, \alpha_1, \alpha_2)$ but these points will be neglect in this study since the parameters β_1 and β_2 have a complex values in both cases.

The Jacobian matrix of the system (2.1) about the first equilibrium state E_1 is

$$J = \begin{bmatrix} -\mu_1 & \alpha_1 & \beta_2 & 0 \\ \alpha_2 & -\mu_2 & 0 & \beta_1 \\ \beta_2 & \beta_1 & \epsilon_1 & 0 \\ \beta_2 & \beta_1 & 0 & \epsilon_2 \end{bmatrix} \tag{2.4}$$

with the characteristic equation

$$s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4 = 0, \tag{2.5}$$

where the coefficients $a_i, (i = 1, 2, 3, 4)$ of the polynomial (2.5) are given by