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Sliding bifurcations and chaos induced by dry friction in a braking system

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Abstract

In this paper, non-smooth bifurcations and chaotic dynamics are investigated for a braking system. A three-degree-of-freedom model is considered to capture the complicated nonlinear characteristics, in particular, non-smooth bifurcations in the braking system. The stick–slip transition is analyzed for the braking system. From the results of numerical simulation, it is observed that there also exist the grazing–sliding bifurcation and stick–slip chaos in the braking system.

1. Introduction

Nonlinear mechanical systems can often be described by smooth dynamical systems. However, for a number of practical engineering systems, the aforementioned description is not always valid. In many engineering applications, dry friction, clearance and impact factors often result in sudden change of the vector fields describing dynamic behaviors of mechanical systems. These systems are not smooth, and are referred to as non-smooth dynamical systems.

Dry friction is a typically non-smooth factor and plays an important role in engineering applications. It is a source of self-sustained oscillations which is called as the stick–slip oscillations. These oscillations often cause some undesired effects observed in engineering applications, which include the noise of a squeaking door, the action of squeal brakes and the motion of mechanical clocks. Therefore, friction oscillators have received a lot of attention from researchers who have provided insightful results. Due to the introduction of new analytical techniques, these non-smooth systems have been studied from the point of view of bifurcation theory in the recent years.

There exists a wide range of research devoted to analysis of low and high dimensional non-smooth systems with dry friction. Popp and Stelter [1] introduced four different models including a single-degree-of-freedom oscillator with external excitation where chaotic behavior characterized by stick–slip motion is found. Moreover, they found different modes of stick–slip behavior and different routes to chaos, which include intermittency and period-doubling in their models. Galvanetto and Knudsen [2] and Galvanetto [3,4] investigated the bifurcations in a two blocks stick–slip system. A one-dimensional map was introduced for studying bifurcations in the four dimensional system. The results observed included a class of bifurcations leading to the onset of stick–slip motions. Awrejcewicz and his collaborators

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[5–7] investigated stick–slip chaos in one-degree-of-freedom and two-degree-of-freedom systems with friction by the Melnikov–Gruendler approach. A chaotic threshold had been obtained for both smooth and stick–slip chaotic behaviors in these systems. Halse et al. [8] examined the behavior of the grazing and sliding bifurcations for the piecewise-smooth one-dimensional maps. Rising phenomena and multi-sliding bifurcations in a two-degree-of-freedom oscillator were investigated by Wagg [9].

An automotive brake device is a typical system with dry friction. The noise of brake systems is an important problem and has received considerable attention from researchers. This attention is due to the economics of the related customer complaints, warranty claims and repairs to disc brake systems, and also due to the difficult nature of the problem, see Refs. [10,11]. Linkaid et al. [12] gave a very detailed review on automotive disc brake squeal. Though these references contain several different theories and a fair amount of controversy, it is thought that brake squeal involves a complicated small amplitude vibration yielded by a frictional force. Many mechanisms have been proposed for disc brake squeal, see Refs. [13–16]. However, few researches on a braking system are done from the point of view of non-smooth dynamics.

The present paper will numerically investigate the complicated dynamics of a six-dimensional autonomous non-smooth system which models a braking system with dry friction. The model was first introduced in [17], and therein simple nonlinear responses were considered. However, non-smooth bifurcations of periodic orbits of the system were not taken into account. We will discuss the aforementioned problem from the point of view of non-smooth dynamics. It is found that there exist very complicated phenomena on non-smooth bifurcations. The organization of the paper is as follows. In Section 2, a three-degree-of-freedom non-smooth dynamical system is considered. Section 3 introduces the theories of non-smooth dynamical systems, in particular, the Filippov system and the sliding bifurcation. The numerical results are given in Section 4. Section 5 contains a discussion of the results.

2. Model

The mechanical system investigated in this paper is shown schematically in Fig. 1, which is a simple model for a braking system. The system contains three mass blocks and an axially moving belt. One of these mass blocks is supported by axially moving belt which models the rotor of a braking system. In the following analysis, we do not consider the deformation of axially moving belt. The V_0 is the velocity of the belt. The first block, m_1, and the second block, m_2, respectively, are connected to a fixed support by a linear spring. The second block is connected with the third block, m_3, by a third linear spring. The contact surface between the third block and the belt is considered to be rough. The third block is pressed on the belt by a normal force F_N. Without loss of the generality, the coefficients of the linear viscous dampers are represented by d_1, d_2 and d_3, respectively.

Let $\widetilde{F}_N = F_N + m_3 g$, where g is the acceleration of gravity. In the following analysis, the maximum static friction force is defined by the usual relation:

$$F_s = \mu_s \widetilde{F}_N, \tag{1}$$

where μ_s is the static friction coefficient.

The dynamic friction coefficient μ_d is expressed by the following relation, which is commonly adopted in Refs. [1–4]:

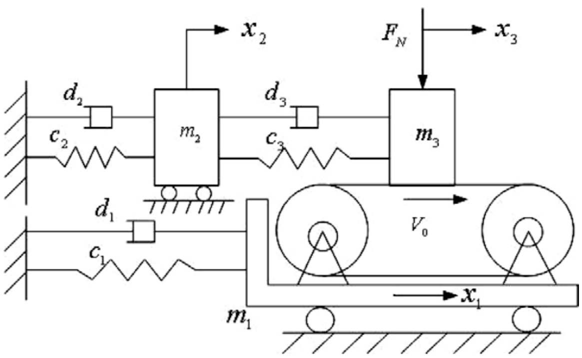

Fig. 1. The model of a braking system.