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Transfer closed and transfer open multimaps in minimal spaces

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Abstract

This paper is devoted to introduce the concepts of transfer closed and transfer open multimaps in minimal spaces. Also, some characterizations of them are considered. Further, the notion of minimal local intersection property will be introduced and characterized. Moreover, some maximal element theorems via minimal transfer closed multimaps and minimal local intersection property are given.

1. Introduction and preliminaries

It is well known that topological concepts have many applications in modern physics. For example, the topology of quantum spacetime is shadowed closely by the Mobius geometry of quasi-Fuschian and Kleinian groups and that is the cause behind the phenomena of high-energy particle physics [11]. In fact, considering the spacetime as the product of two topologies, the topology of space and that of the spacetime will open the way for new line of research in the field of quantum gravity initiated by Witten and El-Naschie. For the relation between Wild Topology, Hyperbolic Geometry and Fusion Algebra with Coupling constants of the standard model and quantum gravity and close connection between ε^{∞} theory and the topological theory of four manifolds, we refer to [12,13] and for the relation between topological concepts and geometrical properties of the ε^{∞} spacetime to [14].

For another example, constructing a topology via a relation on a real-life data will help in mathematizing many fields. In fact, if X is a collection of symptoms and diseases in a certain region and R is a binary relation on X given by an expert the topology on X generated by R is a knowledge base for X, indication of symptoms for a fixed disease can be seen through the topology [7].

Since topology has very important applications in applied sciences, so studying of minimal structure as a generalization of topology is important from this point of view. Minimal structures may have very important applications in quantum particles physics, particularly in connection with string theory and ε^{∞} theory [6,8–10]. The work presented

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in our paper, not initiate new classes with respect to topology but in view of minimal structures. In this paper after introducing transfer closed and transfer open multimaps via minimal space, some characterizations of them are given. Moreover, the concept of minimal local intersection property will be introduced and some its characterization will be investigated. Finally, some maximal element theorems via minimal transfer closed multimaps and minimal local intersection property are considered.

The concepts of minimal structure and minimal spaces, as a generalization of topology and topological spaces were introduced in [21]. For easy understanding of the material incorporated in this paper we recall some basic definitions. For details on the following notions we refer to [1–4,21,22] and [24] and references therein.

A family $\mathscr{M} \subseteq \mathscr{P}(X)$ is said to be a *minimal structure* on X if $\emptyset, X \in \mathscr{M}$. In this case $(X, \mathscr{M})$ is called a *minimal space*. For some examples in this setting see [21,24]. In a minimal space $(X, \mathscr{M})$, $A \in \mathscr{P}(X)$ is said to be an *m-open set* if $A \in \mathscr{M}$ and also $B \in \mathscr{P}(X)$ is an *m-closed set* if $B^c \in \mathscr{M}$. For any $x \in X$, $N(x)$ is said to be a *minimal neighborhood* of x, if for any $z \in N(x)$ there is an m-open subset $G_z \subseteq N(x)$ such that $z \in G_z$. Set $m - Int(A) = \bigcup\{U : U \subseteq A, U \in \mathscr{M}\}$ and $m - Cl(A) = \bigcap\{F : A \subseteq F, F^c \in \mathscr{M}\}$.

Proposition 1.1. [21] *For any two sets A and B,*

(a) *m-$Int(A) \subseteq A$ and m-$Int(A) = A$ if A is an m-open set,*
(b) *$A \subseteq m$-$Cl(A)$ and $A = m$-$Cl(A)$ if A is an m-closed set,*
(c) *m-$Int(A) \subseteq m$-$Int(B)$ and m- $Cl(A) \subseteq m$-$Cl(B)$ if $A \subseteq B$,*
(d) *m-$Int(A \cap B) = (m$-$Int(A)) \cap (m$-$Int(B))$ and $(m$-$Int(A)) \cup (m$-$Int(B)) \subseteq m$-$Int(A \cup B)$,*
(e) *m-$Cl(A \cup B) = (m$-$Cl(A)) \cup (m$-$Cl(B))$ and m -$Cl(A \cap B) \subseteq (m$-$Cl(A)) \cap (m$-$Cl(B))$,*
(f) *m-Int $(m$-$Int(A)) = m$-$Int(A)$ and m–$Cl(m$–$Cl(B)) = m$-$Cl(B)$,*
(g) *$(m$-$Cl(A))^c = m$-$Int(A^c)$ and $(m$-$Int(A))^c = m$-$Cl(A^c)$.*

2. Main Results

Suppose X and Y are two minimal spaces. A *multimap* F: $X \multimap Y$ is a function from a set X into the power set of Y; that is, a function with the values $F(x) \subseteq Y$ for all $x \in X$. Given $A \subseteq X$, set

$$F(A) = \bigcup_{x \in A} F(x).$$

We say that a multimap F: $X \multimap Y$ has a *maximal element* if $F(x_0) = \emptyset$, for some $x_0 \in X$.

For a multimap F: $X \multimap Y$, the multimaps F^c, m–$Cl(F)$, and m–$Int(F)$ from X to Y are defined by $F^c(x) = \{y \in Y$: $y \notin F(x)\}$, $(m$–$Cl(F))(x) = m$–$Cl(F(x))$ and $(m$–$Int(F))(x) = m$–$Int(F(x))$, respectively. Also multimaps F^- and F^* from Y to X are defined by $F^-(y) = \{x \in X$:$y \in F(x)\}$ and $F^*(y) = \{x \in X$:$y \notin F(x)\}$, respectively. Some properties of these multimaps and their relations can be found in [17,19].

Lemma 2.1. [19] *Suppose F, G: $X \multimap Y$ are two multimaps. Then*

(a) *for each $x \in X$, $F(x) \subseteq G(x)$ if and only if $G^*(y) \subseteq F^*(y)$ for each $y \in Y$,*
(b) *$y \notin F(x)$ if and only if $x \in F^*(y)$,*
(c) *for each $x \in X$, $(F^*)^*(x) = F(x)$,*
(d) *for each $x \in X, F(x) \neq \emptyset$ if and only if $\bigcap_{y \in A} F^*(y) = \emptyset$,*
(e) *for each $y \in Y$, $(F^c)^*(y) = F^-(y)$,*
(f) *for each $y \in Y$, $(F^-)^c(y) = F^*(y)$.*

Proposition 2.1. *Suppose F: $X \multimap Y$ is a multimap. Then*

(a) $(m$–$Cl(F))^c = m$–$Int(F^c)$,
(b) $(m$–$Cl(F))^* = (m$– $Int(F^c))^-$,
(c) $(m$–$Cl(F^*))^c = m$–$Int(F^-)$,
(d) $(m$–$Cl(F^*))^* = (m$–$Int(F^-))^-$.

Proof. It is an immediate consequence of Proposition 1.1 and Lemma 2.1. □