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ScienceDirect
Chaos, Solitons and Fractals 40 (2009) 1282–1287
CHAOS SOLITONS & FRACTALS
www.elsevier.com/locate/chaos

Robust tracking control of uncertain Duffing–Holmes control systems

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Accepted 3 September 2007

Abstract

In this paper, the notion of virtual stabilizability for dynamical systems is introduced and the virtual stabilizability of uncertain Duffing–Holmes control systems is investigated. Based on the time-domain approach with differential inequality, a tracking control is proposed such that the states of uncertain Duffing–Holmes control system track the desired trajectories with any pre-specified exponential decay rate and convergence radius. Moreover, we present an algorithm to find such a tracking control. Finally, a numerical example is provided to illustrate the use of the main results.

1. Introduction

In recent years, various chaotic systems have been widely investigated; see, for example [1–16] and the references therein. This is due to theoretical interests as well as to a powerful tool for practical system analysis and control design. Frequently, chaos in many systems is a source of instability and a source of the generation of oscillation. In the past decades, various methodologies in robust control of chaotic system have been proposed, such as variable structure control approach, adaptive sliding mode control approach, adaptive control approach, backstepping control approach, H_∞ control approach, and others.

In this paper, the concept of virtual stabilizability for dynamical systems is firstly introduced and the virtual stabilizability of uncertain Duffing–Holmes control systems is investigated. Using the time-domain approach with differential inequality, a tracking control is proposed such that the states of uncertain Duffing–Holmes control system track the desired trajectories with any pre-specified exponential decay rate and convergence radius. Meanwhile, we present an algorithm to find such a tracking control. A numerical example is also provided to illustrate the use of the main results.

This paper is organized as follows. The problem formulation and main result are presented in Section 2. A numerical example is given in Section 3 to illustrate the main result. Finally, conclusion is made in Section 4.

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0960-0779/$ - see front matter
doi:10.1016/j.chaos.2007.09.007

2. Problem formulation and main result

Nomenclature

$\Re^n$	the n-dimensional real space
$\lvert a \rvert$	the modulus of a complex number a
I	the unit matrix
A^{T}	the transport of the matrix A
$\lVert x \rVert$	the Euclidean norm of the vector $x \in \Re^n$
$\lambda_{\min}(A)$	the minimum eigenvalue of the matrix A with real eigenvalues
$\sigma(A)$	the spectrum of the matrix A
$P > 0$	the matrix P is a symmetric positive definite matrix
$\wedge$	means "and"

Before presenting the problem formulation, let us introduce a lemma which will be used in the proof of the main theorem.

Lemma 1. *If a continuously differentiable real function $s(t)$ satisfies the inequality*

$$\dot{s}(t) \leqslant a - 2\alpha s(t) \quad \forall t \geqslant 0,$$

where $a > 0$ and $\alpha > 0$, then

$$s(t) \leqslant \left[s(0) - \frac{a}{2\alpha} \right] \cdot e^{-2\alpha t} + \frac{a}{2\alpha} \quad \forall t \geqslant 0.$$

Proof. It is easy to deduce that

$$e^{2\alpha t} \cdot \dot{s}(t) + e^{2\alpha t} \cdot 2\alpha s(t) = \frac{d}{dt}[e^{2\alpha t} \cdot s(t)] \leqslant e^{2\alpha t} \cdot a \quad \forall t \geqslant 0.$$

It follows that

$$\int_0^t \frac{d}{dt}[e^{2\alpha t} \cdot s(t)] dt = e^{2\alpha t} \cdot s(t) - s(0) \leqslant \int_0^t e^{2\alpha t} \cdot a\, dt = \frac{a}{2\alpha}(e^{2\alpha t} - 1) \quad \forall t \geqslant 0.$$

Consequently, we have

$$s(t) \leqslant \left[s(0) - \frac{a}{2\alpha} \right] \cdot e^{-2\alpha t} + \frac{a}{2\alpha} \quad \forall t \geqslant 0.$$

This completes the proof. □

In this paper, we consider the following uncertain Duffing–Holmes control systems with uncertain input nonlinearities described as

$$\ddot{x}(t) = -p_1 x - p_2 \dot{x} - x^3 + p_3 \cos(wt) + \Delta f(x, \dot{x}) + \Delta\phi(u, x, \dot{x}), \quad t \geqslant 0, \tag{1a}$$

$$[\, x(0) \quad \dot{x}(0) \,] = [\, x_{01} \quad x_{02} \,], \tag{1b}$$

where $x \in \Re$, $u \in \Re$ is the input, $\Delta f(x, \dot{x})$ represents the plant uncertainty, and $\Delta\phi(u, x, \dot{x})$ represents the uncertain input nonlinearity. For the existence of the solutions of (1), we assume that the unknown terms $\Delta f(x, \dot{x})$ and $\Delta\phi(u, x, \dot{x})$ are all continuous functions. It is well known that the system (1) without any uncertainties (i.e., $\Delta f(x, \dot{x}) = \Delta\phi(u, x, \dot{x}) = 0$) displays chaotic behavior for certain values of the parameters [10]. Letting $x_1 = x$ and $x_2 = \dot{x}$, the corresponding state-space equation of the system (1) is

$$\dot{x}_1 = x_2, \tag{2a}$$

$$\dot{x}_2 = -p_1 x_1 - p_2 x_2 - x_1^3 + p_3 \cos(wt) + \Delta f(x_1, x_2) + \Delta\phi(u, x_1, x_2) \quad \forall t \geqslant 0. \tag{2b}$$

The objective of this paper is to search a tracking control law such that the states x_1 and x_2 of the system (2) track, respectively, the desired trajectories x_d and $\dot{x}_d$. For brevity, let us define the error vector

$$e(t) := [\, e_1(t) \quad e_2(t) \,]^{\mathrm{T}} := [\, x_1(t) - x_d(t) \quad x_2(t) - \dot{x}_d(t) \,]^{\mathrm{T}}. \tag{3}$$

Throughout this paper, the following assumption is made:

(A1) There exist continuous functions $f(x_1, x_2) \geqslant 0$, $r_1(x_1, x_2) \geqslant 0$, and $r_2(x_1, x_2) > 0$, such that, for all arguments