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Anti-synchronization on autonomous and non-autonomous chaotic systems via adaptive feedback control

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Abstract

In this paper, the anti-synchronization of a general class of chaotic systems is investigated. A simple adaptive feedback scheme is proposed to anti-synchronize many familiar chaotic systems, including autonomous systems and non-autonomous systems. Lyapunov analysis for the error system gives the asymptotic stability conditions based on the invariance principle of differential equations. The schemes are successfully applied to three groups of examples: the van der Pol–Duffing oscillator, the parametrically harmonically excited 4D system, and the additionally harmonically excited Murali–Lakshmanan–Chua circuit. Numerical results are presented to justify the theoretical analysis in this paper.

1. Introduction

One of the most striking discoveries in the study of chaos is that chaotic systems can be made to synchronize with others. Nowadays, after the pioneering work of Pecora and Carroll [1], various types of chaos synchronization such as complete synchronization [2,3], phase synchronization [4,5], generalized synchronization [6,7], lag synchronization [8,9], projective synchronization [10,11], noise-induced synchronization [12,13], partially synchronization [14,15], and Q–S synchronization [16,17] have been described.

There is another type of synchronization, anti-synchronization (AS), which is a prevailing phenomenon in symmetrical oscillators [18]. It is well known that the first observation of synchronization of two oscillators by Huygens in the seventeenth century was, in fact, AS between two pendulum clocks, which can also be interpreted as anti-phase synchronization (APS) [19]. That is to say, there is no difference between AS and APS for oscillators with identical amplitudes [20]. But for chaotic systems, that is not the case. As an extension of in coupled phase oscillators, we define AS in chaotic systems as the phenomenon where the variables of two interacting systems have the same amplitude but differ in sign and APS as phase-delayed PS [18]. So far, some progresses have been made in the researches of AS. Kim et al. [21] have found an AS phenomenon in mutually coupled identical Lorenz chaotic systems. Based on a suitable separation of systems, Zhang and Sun [22] have presented some simple but generic criteria for synchronization and anti-synchronization for chaotic systems. Recently, using different control method, the AS for some typical chaotic systems has been discussed [23–25]. However, most of the works mentioned above are designed to anti-synchronize some concrete chaotic

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systems, failing to give a general approach. Moreover, the central dynamical systems considered are mainly autonomous system, i.e., they are not explicitly dependent on time t.

In this paper, we will extend Huang's scheme [26] to study the anti-synchronization for a class of chaotic systems. Firstly, we give the problem formulation of this work in Section 2. The third section is devoted to anti-synchronization for a class of chaotic systems based on adaptive feedback method. Three illustrative examples are considered and numerical simulations are demonstrated in Section 4. Finally, some conclusions are drawn in Section 5.

Note. Throughout the remainder of this paper, the notation $|x|$ represents the absolute value of x, while for $x \in R^n$, $\|x\| = \sqrt{(x^T x)}$ denotes the Euclidean norm of the vector.

2. Problem formulation

Consider a system with a general form, which is usually called as drive system

$$\dot{x} = f(x, t) \tag{1}$$

and a controlled system named as response system

$$\dot{y} = f(y, t) + U(x, y, t), \tag{2}$$

where $x, y \in R^n$ denote the state vectors, and Ω is a domain containing the origin. The function $f(x, t) : \Omega \subset R^n \times R^n \to R^n$ is a smooth nonlinear vector function, which is capable of exhibiting rich dynamics such as chaos and $U(x, y, t)$ is an unknown vector controller.

Let the sate error be $e = y + x$, then the systems (1) and (2) can be recast into

$$\dot{e} = \dot{y} + \dot{x} = f(y, t) + f(x, t) + U(x, y, t), \tag{3}$$

The purpose is to design an adaptive controller to guarantee the error state $e = 0$ is asymptotically stable, i.e. the system (2) anti-synchronizes to the system (1).

Since there are different forms of function $f(x, t)$, we will make in-depth studies mainly in the following cases:

(i) $f(x, t) = f(x)$, i.e. the system we considered is autonomous system;
(ii) $f(x, t) = f_1(x) + G(t)f_2(x) + g(t)$, i.e. the nonlinear function explicitly dependent on time t can be separated from the function implicitly dependent on time t.

Remark 1. Although above-mentioned two forms fail to denote all the nonlinear dynamical system, it can include almost well-known chaotic systems, such as Lorenz-family system, Rössler system, Duffing system, Chua's circuit, hyper-chaotic Rössler system, Duffing–van der Pol system, Duffing–Rayleigh oscillator, and Froude pendulum.

Definition 1. For the two systems described by Eqs. (1) and (2), we say they possess the property of anti-synchronization between $x(t)$ and $y(t)$ if there exists an anti-synchronous manifold (ASM) $M = \{(x(t), y(t)) : x(t) = -y(t)\}$ such that all trajectories $(x(t), y(t))$ approach M as time goes to infinity, that is to say,

$$\lim_{t\to\infty} \|e(t)\| = \lim_{t\to\infty} \|y(t) + x(t)\| = 0.$$

Definition 2. For the two systems described by Eqs. (1) and (2), we say they posses the property of anti-synchronization with tolerance σ (a given small positive constant), if there exists a control function $U(x, y, t)$ such that

$$\lim_{t\to\infty} \|e(t)\| = \lim_{t\to\infty} \|y(t) + x(t)\| \leqslant \sigma.$$

3. Adaptive anti-synchronization

In this section, we would like to present a simple, powerful and adaptive scheme to achieve the anti-synchronization for above-mentioned two types of nonlinear dynamical systems. For this purpose, we have the following assumptions:

Assumption 1. For any $x = (x_1, x_2, \ldots, x_n), y = (y_1, y_2, \ldots, y_n) \in \Omega$, there exists a constant $L > 0$, such that

$$|f_i(x) - f_i(y)| \leqslant L \max_{1\leqslant j\leqslant n} |x_j - y_j|, \quad i = 1, 2, \ldots, n,$$

i.e. the function satisfy the uniform Lipschitz condition and L is the Lipschitz constant.