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On the convexity of fuzzy nets

Kourosh Nourouzi *

Department of Mathematics, K.N. Toosi University of Technology, P.O. Box 16315-1618, Tehran, Iran

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Abstract

In this paper, the convexity properties of fuzzy nets on the Euclidean space $\mathbb{R}^d$ is investigated.

1. Introduction

Convex sets play a key role in quantum logics and quantum information science [2]. For instance, in quantum mechanics and classical theory, the state of a quantum mechanical system forms a convex set [1]. Also, a range of fuzzy values for an event can be expressed as a convex set. A fuzzy interpretation of convexity is that any mixture of two distributions in a set is also in the set.

On the other hand, the notion of fuzzyness has a wide application in many areas of science. In physics, for example, the fuzzy structure of spacetime is followed by the fact that in strong quantum gravity regime spacetime points are determined in a fuzzy manner and therefore the impossibility of determining position of particles gives a fuzzy structure [4,5]. For more applications of fuzzy sets in physics, it is referred to [3,9,12].

In [17], Zadeh paid special attention to the convex fuzzy sets. Studies of convex fuzzy sets were followed by numerous authors [6–8,13,14,16]. In this note, a natural generalization of the concept of fuzzy sets under the name of fuzzy nets is given and then their convexity properties is investigated.

2. Fuzzy nets

Let $\mathbb{R}^d$ denotes the d-dimensional Euclidean space. A fuzzy set in $\mathbb{R}^d$ is a function with domain $\mathbb{R}^d$ and values in the closed interval [0, 1]. For a fuzzy set μ, the subset of $\mathbb{R}^d$ in which μ assumes nonzero values, is known as the support of A (see [17]).

The definition of a convex fuzzy set can be rewritten as follows: the fuzzy set $\mu : \mathbb{R}^d \to [0, 1]$ is said to be convex if

$$\mu(tx + (1-t)y) \geqslant \min\{\mu(x), \mu(y)\},$$

for all $x, y \in \mathbb{R}^d$, and $t \in [0, 1]$. Equivalently, μ is a convex set if and only if the α-level set

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* Fax: +98 21 22853650.
E-mail address: nourouzi@kntu.ac.ir

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$$[\mu]_\alpha = \{x \in \mathbb{R}^d : \mu(x) \geqslant \alpha\}$$

is a convex set for all $\alpha \in [0, 1]$.

Let J be an index set. A J-tuple of elements [0, 1] is a function t: $J \to [0, 1]$. If β is an element of J, we denote the value of t at β by t_β which is βth coordinate of t. We will denote the set of all J-tuples of elements of [0, 1] by $[0, 1]^J$.

Definition 1. For an index set J, a fuzzy net $\mu : \mathbb{R}^d \to [0,1]^J$ on $\mathbb{R}^d$ is defined by $\mu(x) = (\mu_i(x))_{i\in J}$, where each μ_i is a fuzzy set on $\mathbb{R}^d$, and $(\mu_i(x))_{i\in J}$ is a J-tuple.

In case of J is a singleton set, the notions of fuzzy net and fuzzy set are the same, and if $J = \mathbb{N}$ we have a fuzzy sequence ($[0,1]^{\mathbb{N}}$ is the countably infinite cartesian product of [0, 1] with itself). The set of all fuzzy nets on $\mathbb{R}^d$ will be denoted by $\mathscr{FN}(\mathbb{R}^d)$.

Throughout of this section J stands for any index set, and ϑ will be a fuzzy set valued function $\vartheta : \mathscr{FN}(\mathbb{R}^d) \to [0,1]^{\mathbb{R}^d}$ which maps each element μ in $\mathscr{FN}(\mathbb{R}^d)$ to a fuzzy set $\vartheta(\mu)$ on $\mathbb{R}^d$.

Example 1. In each the following cases ϑ is a function which maps every element $\mu = (\mu_i)_{i\in J} \in \mathscr{FN}(\mathbb{R}^d)$ to a fuzzy set on $\mathbb{R}^d$.

(i) $\vartheta(\mu) = \sup_i \mu_i$, $\vartheta(\mu) = \inf_i \mu_i$, $\vartheta(\mu) = \limsup_i \mu_i$, and $\vartheta(\mu) = \liminf_i \mu_i$.
(ii) (The projection map) $\vartheta(\mu) = \vartheta_i(\mu) = \mu_i$.

Example 2. *Special implication nets* on $[0, 1]^J$
Let $a = (a_i)$, $b = (b_i) \in [0, 1]^J$.

1. *Zadeh* implication net
$$N_Z(a,b) = ((1 - a_i) \vee (a_i \wedge b_i))_{i\in J}.$$
2. *Lukasiewicz* implication net
$$N_{Lu}(a,b) = ((1 - a_i + b_i) \wedge 1)_{i\in J}.$$
3. *Mamdani* implication net
$$N_M(a,b) = (a_i \wedge b_i)_{i\in J}.$$
4. *Gaines–Rescher* implication net
If
$$c_i = \begin{cases} 1 & a_i \leqslant b_i, \\ 0 & a_i > b_i, \end{cases}$$
then
$$N_{\mathrm{GR}}(a,b) = (c_i)_{i\in J}.$$
5. *Gödel* implication net
If
$$c_i = \begin{cases} 1, & a_i \leqslant b_i, \\ b_i, & a_i > b_i, \end{cases}$$
then
$$N_G(a,b) = (c_i)_{i\in J}.$$
6. *Goguen* implication operator
If
$$c_i = \begin{cases} 1, & a_i = 0, \\ \frac{b_i}{a_i}, & a_i > b_i, \end{cases}$$
then