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A multiparameter chaos control method based on OGY approach

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Abstract

Chaos control is based on the richness of responses of chaotic behavior and may be understood as the use of tiny perturbations for the stabilization of a UPO embedded in a chaotic attractor. Since one of these UPO can provide better performance than others in a particular situation the use of chaos control can make this kind of behavior to be desirable in a variety of applications. The OGY method is a discrete technique that considers small perturbations promoted in the neighborhood of the desired orbit when the trajectory crosses a specific surface, such as a Poincaré section. This contribution proposes a multiparameter semi-continuous method based on OGY approach in order to control chaotic behavior. Two different approaches are possible with this method: coupled approach, where all control parameters influences system dynamics although they are not active; and uncoupled approach that is a particular case where control parameters return to the reference value when they become passive parameters. As an application of the general formulation, it is investigated a two-parameter actuation of a nonlinear pendulum control employing coupled and uncoupled approaches. Analyses are carried out considering signals that are generated by numerical integration of the mathematical model using experimentally identified parameters. Results show that the procedure can be a good alternative for chaos control since it provides a more effective UPO stabilization than the classical single-parameter approach.

1. Introduction

A chaotic attractor has a dense set of unstable periodic orbits (UPOs) and the system often visits the neighborhood of each one of them. Moreover, chaotic response has sensitive dependence to initial condition, which implies that the system's evolution may be altered by small perturbations. Chaos control explores all this richness employing tiny perturbations for the stabilization of a UPO embedded in a chaotic attractor. This makes this kind of behavior to be desir-

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able in a variety of applications, since one of these UPO can provide better performance than others in a particular situation.

Chaos control methods may be classified as discrete or continuous techniques. The first chaos control method was proposed by Ott et al. [9], nowadays known as the OGY method as a tribute of their authors (Ott–Grebogi–Yorke). This is a discrete technique that considers small perturbations applied in the neighborhood of the desired orbit when the trajectory crosses a specific surface, such as some Poincaré section [5,16]. On the other hand, continuous methods are exemplified by the so called delayed feedback control, proposed by Pyragas [12], which states that chaotic systems can be stabilized by a feedback perturbation proportional to the difference between the present and a delayed state of the system. There are many improvements of the OGY method that aim to overcome some of its original limitations, as for example: control of high periodic and high unstable UPO [6,8,13], control using time delay coordinates [4,7,17] and control using multiparameter approach based on pole placement formalism [2].

The semi-continuous (SC) control method lies between the continuous and the discrete time control because one can introduce as many intermediate Poincaré sections, viewed as control stations, as it is necessary to achieve stabilization of a desirable UPO. Therefore, the SC method provides a more effective control since it allows a great number of actuation during a period.

This contribution considers a semi-continuous multiparameter chaos control method built upon the OGY method [10,11,14]. The idea is to use different control parameters in order to perform the UPO stabilization and, because of that, the map that establishes the relation between the system responses in two subsequent control stations depends on all control parameters. The proposed method assumes that only one control parameter actuates in each control station, defining active and passive parameters. On this basis, two different approaches may be adopted for the general formulation: coupled and uncoupled approaches. The coupled approach considers that all control parameters influence the system dynamics although they are not active. The uncoupled approach, on the other hand, is a particular case where control parameters return to the reference value when they become passive and therefore, they are not influencing the system dynamics. As an application of the general formulation a two-parameter control of a nonlinear pendulum is carried out considering coupled and uncoupled approaches. All signals are numerically generated by the integration of the mathematical model equations, using experimentally identified parameters. The close-return (CR) method [1] is employed to determine the UPO embedded in the attractor. Afterwards, the local dynamics expressed by the Jacobian matrix and the sensitivity matrix of the transition maps in a neighborhood of the control points are determined using the least-square fit method [1,8,10,11,14]. Moreover, the singular value decomposition (SVD) technique is employed for determining the stable and unstable directions near the control point. Results show that the multiparameter approaches can be good alternatives for chaos control since it provides a more effective UPO stabilization when compared to those obtained from the classical single parameter approach.

2. Multiparameter chaos control method

A chaos control method may be understood as a two stage technique. The first step is known as learning stage where the unstable periodic orbits are identified and some system characteristics are evaluated. After that, there is the control stage where the desirable UPOs are stabilized.

The OGY approach is described considering a discrete system of the form of a map $\xi^{n+1} = F(\xi^n, p)$, where $p \in \Re$ is an accessible parameter for control. This is equivalent to a parameter dependent map associated with a general surface, usually a Poincaré section. The control idea is to monitor the system dynamics until the neighborhood of a desirable point is reached. After that, a proper small change in the parameter p causes the next state ξ^{i+1} to fall into the stable direction of the desirable point. In order to find the proper variation in the control parameter, δp, it is considered a linearized version of the dynamical system near this control point. The linearization has a homeomorphism with the nonlinear problem that is assured by the Hartman–Grobman theorem [9,15]. The semi-continuous control method introduces as many intermediate control stations as it is necessary to achieve stabilization of a desirable UPO. In order to use N control stations per forcing period T, one introduces N equally spaced successive Poincaré sections Σ_n $(n = 1, \ldots, N)$.

The multiparameter (MP) chaos control method considers N_{p} different control parameters, p_i $(i = 1, \ldots, N_{\mathrm{p}})$. By considering a specific control station, only one of those control parameters actuates. Under this assumption, the map F, that establishes the relation of the system behavior between the control stations Σ_n and Σ_{n+1}, depends on all control parameters. Although only one parameter actuates in each section, it is assumed the influence of all control parameters based on their positions in station Σ_n. On this basis

$$\xi^{n+1} = F(\xi^n, P^n) \tag{1}$$