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Modified homotopy perturbation method for solving non-linear Fredholm integral equations

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Abstract

A numerical solution for solving non-linear Fredholm integral equations is presented. The method is based upon homotopy perturbation theory. The result reveal that the modified homotopy perturbation method (MHPM) is very effective and convenient.

1. Introduction

A new perturbation method called homotopy perturbation method (HPM) was proposed by *He* in 1997 and systematical description in 2000 which is, in fact, a coupling of the traditional perturbation method and homotopy in topology [1]. This new method was further developed and improved by *He* and applied to non-linear oscillators with discontinuities [2], non-linear wave equations [3], asymptotology [4], boundary value problem [5], Limit cycle and bifurcation of non-linear problems [6] and many other subjects. Thus *He*'s method is a universal one which can solve various kinds of non-linear equations. For example, it was applied to the quadratic Ricatti differential equation by abbasbandy [7]; to the axisymmetric flow over a stretching sheet by Ariel et al. [8]; to the non-linear systems of reaction-diffusion equations by Ganji et al. [9]; to the Helmholtz equation and fifth-order KdV equation by Rafei et al. [10]; for the thin film flow of a fourth grade fluid down a vertical cylinder by Siddiqui et al. [11]; to the non-linear Voltra–Fredholm integral equations by Ghasemi et al. [12]. Recently various powerful mathematical methods such as variational iteration method [13–21], Exp-function method [22,23], F-expansion method [24], Adomian decomposition method [25] and others [26,27] have been proposed to obtain exact and approximate analytic solutions for linear and non-linear problems. This paper, applies the modified homotopy perturbation method [1–12] to the discussed problem.

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2. Modified homotopy perturbation method

Consider the following non-linear Fredholm integral equation

$$U(x) = G(x) + \int_0^1 K(x,t)T(U(t))\mathrm{d}t, \quad 0 \leqslant x \leqslant 1, \tag{1}$$

where

$$K(x,t) = g(x)h(t).$$

In Eq. (1) the functions K, G and T are given, and U the solution to be determined. We assume that (1) has the unique solution. To explain MHPM, we consider (1) as

$$L(U) = U(x) - G(x) - \int_0^1 K(x,t)T(U(t))\mathrm{d}t, \tag{2}$$

with solution $U(x)$. We can define homotopy $H(U,p,M)$ by

$$H(U,0,M) = F(U), \quad H(U,1,M) = L(U),$$

where M is an unknown real number and

$$F(U) = U(x) - G(x). \tag{3}$$

Typically we may choose a convex homotopy by

$$H(U,p,M) = (1-p)F(U) + pL(U) + p(1-p)[Mg(x)] = 0, \tag{4}$$

where M is called the accelerating parameters, and for $M = 0$ we define $H(U,p,0) = H(U,p)$, which is the standard HPM.

The convex homotopy (4) continuously trace an implicitly defined curve from a starting point $H(U(x) - G(x),0,M)$ to a solution function $H(U(x),1,M)$. The embedding parameter p monotonically increases from zero to unit as trivial problem $F(U) = 0$ is continuously deformed to original problem $L(U) = 0$.

The MHPM uses the homotopy parameter p as an expanding parameter to obtain [3]:

$$V = U_0 + pU_1 + p^2U_2 + \cdots, \tag{5}$$

when $p \to 1$, (4) corresponds to the original one, (5) becomes the approximate solution of Eq. (1), i.e.,

$$U = \lim_{p \to 1} V = U_0 + U_1 + U_2 + \cdots \tag{6}$$

Substituting (3) and (5) into (4) results into

$$(1-p)(U_0 + pU_1 + p^2U_2 + \cdots - G(x)) + p(U_0 + pU_1 + p^2U_2 + \cdots - G(x) - g(x) \times \int_0^1 h(t)T(U_0 + pU_1 + p^2U_2 + \cdots)\mathrm{d}t) + p(1-p)[Mg(x)] = 0. \tag{7}$$

In Eq. (7) we can write $T(U_0 + pU_1 + p^2U_2 + \cdots)$ as follows

$$T(U_0 + pU_1 + p^2U_2 + \cdots) = A_0 + pA_1 + p^2A_2 + \cdots, \tag{8}$$

where A_k are Adomian polynomials [28] which depend upon $U_0, U_1, \ldots, U_k$. By differentiating both sides of Eq. (8) we can write

$$\frac{\mathrm{d}^k}{\mathrm{d}p^k} T(U_0 + pU_1 + p^2U_2 + \cdots)|_{p=0} = \frac{\mathrm{d}^k}{\mathrm{d}p^k}(A_0 + pA_1 + p^2A_2 + \cdots)|_{p=0}. \tag{9}$$

From Eq. (9) we conclude that

$$A_k = A_k(U_0, U_1, \ldots, U_k) = \frac{1}{k!}\frac{d^k}{dp^k} T(U_0 + pU_1 + p^2U_2 + \cdots)|_{p=0}, \quad k = 0, 1, \ldots \tag{10}$$

Substitution (8) into (7) result into

$$(1-p)(U_0 + pU_1 + p^2U_2 + \cdots - G(x)) + p(U_0 + pU_1 + p^2U_2 + \cdots - G(x) - g(x)\int_0^1 h(t)[A_0 + pA_1 + p^2A_2 + \cdots]\mathrm{d}t) + p(1-p)[Mg(x)] = 0. \tag{11}$$