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A note on synchronization between two different chaotic systems

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Abstract

In this paper, a new control method based on the Lyapunov method and linear matrix inequality framework is proposed to design a stabilizing controller for synchronizing two different chaotic systems. The feedback controller is consisted of two parts: linear dynamic control law and nonlinear control one. By this control law, the exponential stability for synchronization between two different chaotic systems is guaranteed. As applications of proposed method, synchronization problem between Genesio–Tesi system and Chen system has been investigated, and then the similar approach is applied to the synchronization problem between Rössler system and Lorenz system.

1. Introduction

Chaos is very interesting nonlinear phenomenon and has applications in many areas such as biology, economics, signal generator design, secure communication, many other engineering systems, and so on. Since the pioneering work of Pecora and Carroll [1], chaos synchronization has become an active research topic in nonlinear science because of its extensive applications. Different types of synchronization problems have been observed and studied in various chaotic/hyper-chaotic systems, such as identical synchronization, phase synchronization, lag synchronization, generalized synchronization and so on [2–19]. Most of theoretical results about synchronization phenomena focus on structurally equivalent systems. However, we often need to discuss coherent behavior of strictly different dynamical systems. In fact, in systems such as laser array, biological systems to cognitive processes, it is hardly the case that every component can be assumed to be identical. Consequently, in these years, more and more applications of chaos synchronization in secure communications make it much more important to synchronize two different chaotic systems [20]. On the other hand, as is well-known, fast convergence of a system is essential for real-time computation. In the same manner, it is required for the chaotic systems that they are not only synchronized but also with a fast synchronizing rate. Thus, exponential synchronization for various chaotic systems has been also investigated [21–23] in very recent years.

This paper considers the exponential synchronization of two different chaotic systems. A novel control scheme, which is consisted of a linear dynamic feedback controller and a nonlinear active feedback one, has been proposed.

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Then, based on the Lyapunov stability theory and a recent result [24], the controller existence criterion for exponential synchronization of the systems is derived in terms of LMIs, which can be easily solved by various convex optimization algorithms developed recently [26].

Through the paper, $\mathscr{R}^n$ denotes n-dimensional Euclidean space, and $\mathscr{R}^{n\times m}$ is the set of all $n\times m$ real matrices. $X<0$ means that X is a real symmetric negative definitive matrix. I denotes the identity matrix with appropriate dimensions. $\|\cdot\|$refers to Euclidean vector norm or the induced matrix 2-norm. The organization of this paper is as follows. In Section 2, the problem statement and controller design method are presented. In Section 3, we provide two applications to demonstrate the usefulness of the proposed method. Finally concluding remark is given.

2. Main results

Consider a class of chaotic system described by

$$\dot{x} = Ax + f(x), \tag{1}$$

where $x \in \mathscr{R}^n$ is the state vector, $A \in \mathscr{R}^{n\times n}$ and $f(x) : \mathscr{R}^n \to \mathscr{R}^n$ are the linear coefficient matrix and nonlinear part of system (1), respectively.

Eq. (1) is considered as drive system and the response system with control inputs is introduced as follows:

$$\dot{y} = By + g(y) + Hu + \alpha, \tag{2}$$

where $y \in \mathscr{R}^n$ is the state vector, $B \in \mathscr{R}^{n\times n}$ and $g(x) : \mathscr{R}^n \to \mathscr{R}^n$ are the linear coefficient matrix and continuous nonlinear vector function, $H \in \mathscr{R}^{n\times p}$ is the input matrix, and $u \in \mathscr{R}^p$ and $\alpha \in \mathscr{R}^n$ are the linear dynamic feedback control law and nonlinear static feedback one, respectively.

The purpose of chaos synchronization is how to design the controllers, which are able to synchronize the states of both the drive and the response systems. If we define the error vector as $e = y - x$, the dynamic equation of synchronization error can be expressed as

$$\dot{e} = Be + (B - A)x + g - f + Hu + \alpha. \tag{3}$$

Hence, the objective of synchronization is to make

$$\lim_{t\to\infty} \|e(t)\| = 0.$$

In order to synchronize two different chaotic systems, first we design the control input α using active control method:

$$\alpha = -(B - A)x - g + f. \tag{4}$$

Note that the active control method is the widely used method for chaotic synchronization in the literature.

Then the controlled error system is simplified as

$$\dot{e} = Be + Hu. \tag{5}$$

Now, let us consider the control input u for stability of error system. For this end, we propose the following linear dynamic controller:

$$\begin{aligned} \dot{\xi}(t) &= \mathscr{A}_c\xi(t) + \mathscr{B}_c e(t), \\ u(t) &= \mathscr{C}_c\xi(t), \quad \xi(0) = 0, \end{aligned} \tag{6}$$

where $\xi(t) \in \mathscr{R}^n$ is the controller state, and $\mathscr{A}_c, \mathscr{B}_c$, and $\mathscr{C}_c$ are gain matrices with appropriate dimensions to be determined later.

Applying this controller (4)–(6) to system (5) results in the closed-loop system

$$\dot{z}(t) = \bar{A}z(t), \tag{7}$$

where

$$z(t) = \begin{bmatrix} e(t) \\ \xi(t) \end{bmatrix}, \quad \bar{A} = \begin{bmatrix} B & H\mathscr{C}_c \\ \mathscr{B}_c & \mathscr{A}_c \end{bmatrix}.$$

Definition 1. If there exist a positive constant k and $\gamma > 0$ such that

$$\|e(t)\| \leqslant \gamma \mathrm{e}^{-kt} \quad \forall t > 0,$$