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Numerical investigation of the instability of Benard problem

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Abstract

In this work, the dynamics of instability of a liquid layer heated from below, which is know as Benard problem is investigated. It is a prototype of nonlinear problem where the instability is governed by the two parameters: the Grashof number *Gr*, and the Prandtl number *Pr*.

To shed some light on the instability of the problem and to understand the route to chaos, a small perturbation was introduced to the flow field using a sinusoidal function with small amplitude. The effect of this perturbation was then studied by changing the amplitude regularly.

Finite difference method was employed to solve numerically the associated system of partial differential equations. Results of these calculations were analyzed using the modern theory of dynamical systems.

Numerical results indicate that for fixed values of the two parameters: *Pr* and *Gr* and for relatively large values of the amplitude the system will become chaotic. Numerical results indicate that the system will become chaotic through intermittency.
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1. Introduction

The Bénard convection problem has been extensively studied experimentally and theoretically because of its frequent occurrence in various fields of science and engineering. A full account of the linearized theory is given in [1,2]. Lord Rayleigh (1916) (see [15 and the references therein]) made the first theoretical analysis of this problem concerning the stability of a fluid layer in the presence of a temperature gradient parallel to the gravitational force. The linear theory predicts only the onset of instabilities and determines the critical Rayleigh number and wavenumber, it does not predicts the intensity of convection for a given Rayleigh number. Once the flow becomes unstable, the initially small disturbance will grow with time, and the subsequent fluid motion will be governed by the nonlinear Navier–Stockes equation.

These analyses assume that the flow and temperature fields are periodic in the horizontal directions and seek normal mode solutions so that the resulting equation for the hydrodynamic stability analysis become one-dimensional, which can be solved even analytically. Later, to make the analysis more compatible with experiments, some investigators try to consider the effects of lateral walls on the flow pattern and size of convection cells numerically as well as theoretically. Davis [3] was the first investigator to study the linear hydrodynamic stability of the Rayleigh–Bénard convection in a

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fully confined domain numerically. Reddy and Voyè [4] and van de Vooren and Dijkstra [5] employed a finite element method to analyze linear convective instability in finite domains.

There have been also other attempts to consider the effects of confining sidewalls employing multi-scale perturbation theory. In a seminal paper Segel [6] investigated the consequences of the presence of vertical lateral walls in a rectangular container on the onset and the amplitude of convection and derived a Ginzburg–Landau type equation. This kind of analysis was later employed by other investigators [7] to study the effect of a small heat transfer through the sidewalls, which makes the onset of convection an imperfect bifurcation. The Ginzburg–Landau equation derived through the multi-scale perturbation method has been extensively employed in the study of pattern formation in the Rayleigh–Bénard convection [8]. Nonlinear analysis in a vertical cylinder has been done by Hardin and Sani [9]. All these analyses until now are related with the Rayleigh–Bénard convection in rectangular, cylindrical or spherical domains where the governing equations are separable in the corresponding coordinate system.

The intensity of thermal convection and the critical Rayleigh number depend crucially on the shapes of the domain. Therefore the shape of the convection domain may be considered as an important control parameter in adjusting convection. For example, a judicious design of the sidewall shapes can suppress or enhance the natural convection in the vessel. Until now, there were no appropriate analysis tools for this interesting problems of hydrodynamic stability. But recently a method of linear and nonlinear hydrodynamic stability analysis in confined domains with nonslip walls was employed [10] by exploiting the Chebyshev pseudospectral method [11]. Mihir et al. [13] studied the principle of exchange instability in the hydrodynamic simple Benard problem. Mulone and Rionero [14] studied the nonlinear stability of the rotating Benard problem using Lyapunov direct method.

Chen et al. [15], Gao et al. [16] and Zhou et al. [17] studied the relation between the Bednard problem and the Lorenz system. Chen in [15] was able to extend the Lorenz system to be presented by a system of five ordinary differential equations where the Lorenz attractor is the projection of the five dimensional solution into three-dimensional coordinates x, y and z.

In the present investigation, we use numerical techniques to solve the nonlinear hydrodynamic stability problems of the Rayleigh–Bénard convection in two-dimensional finite domains of rectangular shape. We reformulate the Navier–Stokes equations using the stream function and the vorticity function. Then the finite difference method is employed to solve the associated set of partial differential equations.

The paper will be organized as follows. The mathematical formulation of the problem will be considered in the next section, and the numerical results will be discussed in Section 3 and conclusion remarks will be presented in Section 4.

2. Mathematical formulation of the problem

We consider a two-dimensional flow on the x–y plane contained between two flat plates at $y = 0$ and $y = H$, respectively. The fluid originally at a uniform temperature To is heated from below by increasing the temperature of the lower plate suddenly to T_1, then the density of the fluid at any location becomes smaller than the fluid just above it. If a fluid parcel is displaced slightly upward into the region of higher density, a buoyant force will assist it to move further upward. Similarly, if the fluid parcel is displaced downward into a region of smaller density, it will keep moving in the same direction. Without a sufficiently large, viscous, retarding force, this situation is said to be unstable, and the instability appears in the form of a net of hexagonal convections cells. The fluid motion under consideration is governed by the following set of equations:

The continuity equation:

$$\frac{\partial\rho}{\partial t} + \Delta\cdot(\rho V) = 0, \tag{2.1}$$

the Navier–Stokes equation

$$\rho\frac{\mathrm{D}V}{\mathrm{D}t} = -\Delta p - \Delta\times[\mu(\Delta\times V)] + \Delta[(2\mu+\lambda)\Delta\cdot V], \tag{2.2}$$

the energy equation

$$\rho\frac{\mathrm{D}h}{\mathrm{D}t} = \frac{\mathrm{D}p}{\mathrm{D}t} + \Delta\cdot[\kappa\Delta T] + \Phi, \tag{2.3}$$

and the state equation

$$\rho = \rho_0[1-(T-T_0)], \tag{2.4}$$

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