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Stability criteria for a class of differential inclusion systems with discrete and distributed time delays

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Abstract

In this paper, the global asymptotic stability for a class of differential inclusion systems with discrete and distributed time delays is investigated. Some delay-dependent criteria are proposed to guarantee the global asymptotic stability of the systems. Finally, a numerical example is provided to illustrate the use of the main results.

1. Introduction

Recently, much effort has focused on the stability criteria and control design of differential inclusion systems; see, for example, [1–6] and the references therein. Generally speaking, any dynamic systems can be represented as the differential inclusion equations when the uncertainties exist in such dynamic systems. On the other hand, any physical system inherently owns, more or less, some time-delay phenomena because the energy's propagation of the dynamic system is with a finite speed; see, for example, [5,7–19]. Frequently, the delay in many systems is a source of instability and a source of the generation of oscillation. Such systems exist in various fields of application, such as economics, biology, physicals, engineering, and medicine. For physical and engineering systems, these include flip–flop circuits, communications systems, nuclear reactors, rocket engines, the ship stabilization, the manual control, the microwave oscillator, transistor design, and chemical engineering systems. Therefore the robust stability of differential inclusion time-delay systems is due not only to theoretical interests but also to an effective tool for the robust stability analysis and control design. The reasoning of the stability criterion for differential inclusion time-delay systems is in general not as easy as that without time delays. The stability criteria for differential inclusion time-delay systems can be classified into two categories, namely delay-dependent criteria and delay-independent criteria. Generally speaking, the latter ones are more conservative than the former ones, since the latter ones must work for any delay. It is the purpose of this paper to investigate the delay-dependent criteria for the global asymptotic stability of differential inclusion time-delay systems. In this paper, based on the frequency-domain approach, some delay-dependent criteria are proposed to guarantee the global asymptotic stability for a class of differential inclusion systems with discrete and distributed time delays.

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Nomenclature

$\Re^n$	the n-dimensional real space
$\Re^{m\times n}$	the set of all real m by n matrices
$\|a\|$	the modulus of a complex number a
I	the unit matrix
A^{-1}	the inverse of the matrix A
$\\|x\\|$	the Euclidean norm of the vector $x \in \Re^n$
$\\|A\\|$	the induced Euclidean norm (or spectral norm) of the matrix A
$r(A)$	the spectral radius of the matrix A
$\det(A)$	the determinant of the matrix A
$\lambda_i(A)$	the ith eigenvalue of the matrix A
$\mathrm{Re}[s]$	the real part of the complex number s
$\|A\|$	$[\|a_{rs}\|]$ with $A=[a_{rs}]$
$A \leqslant B$	iff $a_{rs} \leqslant b_{rs}$ for all r, s', with $A=[a_{rs}]$ and $B=[b_{rs}]$
$\sigma_{\max}(A)$	the maximum singular value of A
$\\|H(s)\\|_\infty$	$\sup_w \sigma_{\max}[H(jw)]$
$\underline{q}$	$\{1,2,\cdots,q\}$
$\bar{q}$	$\{0,1,2,\cdots,p\}$

This paper is organized as follows. In Section 2, some delay-dependent criteria are proposed to guarantee the global asymptotic stability for a class of differential inclusion systems with discrete and distributed time delays. An example is given in Section 3 to illustrate the main results. Section 4 presents the conclusions.

2. Problem formulation and main results

Before presenting the problem formulation, let us introduce some lemmas which will be used in the proofs of the main theorems.

Lemma 1 [20]. *If the matrices A and $B \in \Re^{n\times m}$ satisfy $|A| \leqslant B$, then $\|A\| \leqslant \|B\|$.*

Lemma 2 [21]. *Let $A \in \Re^{n\times n}$, then we have $r(A) \leqslant \|A\|$.*

Lemma 3. *If the matrix $A \in \Re^{n\times n}$ satisfies $\|A\| < 1$, then* $\det[I-A]\neq 0$.

Proof 1. By Lemma 2, it can be obtained that $r(A) \leqslant \|A\| < 1$, which implies that $1 \notin \{\lambda_i(A)|i \in \underline{n}\}$. Thus, it can be deduced that $\det[I-A]\neq 0$. This completes the proof. □

Lemma 4 [22]. *Suppose that $f(z)$ is a nonconstant analytic function on any region R. Then $f(z)$ cannot attain its maximum modulus at an interior point of R.*

In this section, we consider the following differential inclusion systems with discrete and distributed time delays

$$\dot{x}(t) \in A_{I,0}x(t) + \sum_{i=1}^{q} A_{I,i}x(t-h_i) + \sum_{i=1}^{q} \int_{t-d_i}^{t-c_i} B_{I,i}x(s)\mathrm{d}s, \tag{1a}$$

$$x(t) = \varphi(t), \quad t \in [-H, 0], \tag{1b}$$

where $x \in \Re^n$ is the state vector, $h_i's$, $c_i's$, and $d_i's$ are constant delays with $0 \leqslant c_i \leqslant d_i$, $\forall i \in \underline{q}$, $H := \max\{h_i, c_i, d_i | i \in \underline{q}\}$, $\varphi(t)$ is a given continuous vector-valued initial function, and