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Two classes of stacked central configurations for the spatial $2n+1$-body problem: Nested regular polyhedra plus one

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ABSTRACT

In this paper we consider $2n$ mass points located at the vertices of two nested regular polyhedra with the same number of vertices and the $(2n+1)$th mass located at the geometrical center of the nested regular polyhedra. We show the existence of central configurations for any given mass ratios and the size ratio of nested polyhedra.

1. Introduction and main results

The classical n-body problem concerns with the motion of n mass points moving in space according to the Newtonian law:

$$m_i\ddot{x}_i = \sum_{k=1,k\neq i}^{n} \frac{m_k m_i (x_k - x_i)}{r_{ki}^3}, \quad i = 1, \ldots, n. \tag{1.1}$$

Here the gravitational constant equals to one, $x_i \in \mathbb{R}^d (1 \leq d \leq 3)$ is the position of mass $m_i > 0$ and $r_{ki} = |x_k - x_i|$ is the Euclidean distance between x_k and x_i.

Let $M = m_1 + \cdots + m_n$ be the total mass and

$$c = \frac{1}{M}(m_1x_1 + \cdots + m_nx_n)$$

be the center of mass of the configuration $x = (x_1, \ldots, x_n)$. The space of configuration is defined by

$$X = \{x \in (\mathbb{R}^d)^n : c = 0, x_i \neq x_j \text{ for all } i \neq j\}.$$

A configuration $x = (x_1, \ldots, x_n) \in X$ is called a *central configuration* if there exists some positive constant λ, called the multiplier, such that

$$-\lambda x_i = \sum_{j=1,j\neq i}^{n} \frac{m_j(x_j - x_i)}{r_{ij}^3}, \quad i = 1, \ldots, n. \tag{1.2}$$

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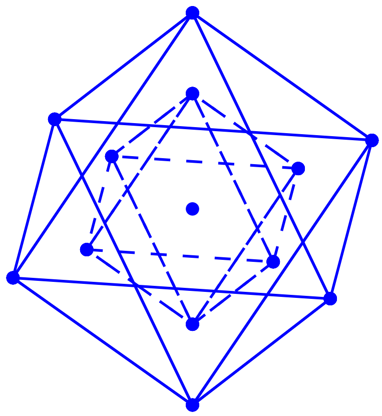

Fig. 1. Nested regular octahedra plus one.

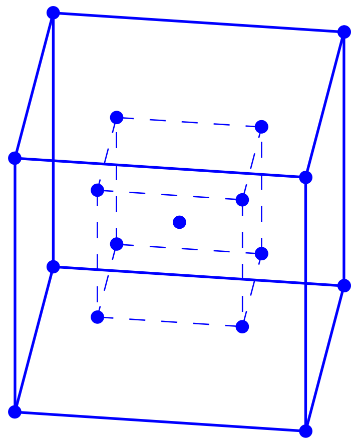

Fig. 2. Nested regular cube plus one.

It is easy to see that a central configuration remains a central configuration after a rotation in $\mathbb{R}^d$ and a scalar multiplication. More precisely, let $A \in SO(d)$ and $a > 0$, if $x = (x_1, \dots, x_n)$ is a central configuration, so are $Ax = (Ax_1, \dots, Ax_n)$ and $ax = (ax_1, \dots, ax_n)$.

Two central configurations are said to be equivalent if one can be transformed to the other by a scalar multiplication and a rotation. In this paper, when we say a central configuration, we mean a class of central configurations as defined by the above equivalence relation.

The study of central configuration goes back to Euler and Lagrange. For $n = 3$, it is a classical result that there are three collinear, called Euler, central configurations and one equilateral triangular, called Lagrange, central configurations. For $n = 4$, Moulton [1] proved that there is exactly one collinear central configuration for each arrangement of the mass points on the line.

There are several reasons why central configurations are of special importance in the study of the n-body problem; see [2–4] for details.

A *stacked central configuration* is a central configuration in which a proper subset of the n bodies is already in a central configuration. This class of central configuration of 5-body problem was introduced by Hampton in [5]. The work of [5] was complemented by Llibre in [6,7].

In this paper we are interested in spatial central configurations, that is $d = 3$. Zhang and Zhou [8] showed the existence of double pyramidal central configurations of $N + 2$-body problem. The authors [9–11] provided some examples of stacked central configurations for the spatial 7-body problem.

The authors [12] showed the existence of spatial central configurations of nested regular polyhedra. In this paper we consider $2n$ masses located at the vertices of two nested regular polyhedra with the same number of vertices and the $(2n + 1)$th mass located at the geometrical center of the nested regular polyhedra (see Figs. 1–2). The case $n = 4$ can be found in [13]. The stacked nested regular octahedra(cube) plus one central configurations are characterized in the following sections.