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Review

The gauging of BV algebras

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ABSTRACT

A BV algebra is a formal framework within which the BV quantization algorithm is implemented. In addition to the gauge symmetry, encoded in the BV master equation, the master action often exhibits further global symmetries, which may be in turn gauged. We show how to carry this out in a BV algebraic set up. Depending on the nature of the global symmetry, the gauging involves coupling to a pure ghost system with a varying amount of ghostly supersymmetry. Coupling to an $N = 0$ ghost system yields an ordinary gauge theory whose observables are appropriately classified by the invariant BV cohomology. Coupling to an $N = 1$ ghost system leads to a topological gauge field theory whose observables are classified by the equivariant BV cohomology. Coupling to higher N ghost systems yields topological gauge field theories with higher topological symmetry. In the latter case, however, problems of a completely new kind emerge, which call for a revision of the standard BV algebraic framework.

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1. Introduction

The Batalin–Vilkovisky (BV) approach [1,2] is the most general and powerful quantization algorithm presently available. It is suitable for the quantization of ordinary gauge theories, such as Yang–Mills theory, as well as more complicated gauge theories with open and/or reducible gauge symmetries. Its main feature consists in the introduction of ghost fields from the outset automatically incorporating in this way BRST symmetry.

The general structure of the BV formalism is as follows [3,4]. Given a classical field theory with gauge symmetries, one introduces an antifield with opposite statistics for each field, including ghost fields, therefore doubling the total number of

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fields. The resulting field/antifield space $\mathcal{F}$ is equipped with an odd Poisson bracket $\{\cdot,\cdot\}$, called an antibracket, and acquires an odd phase space structure, in which fields and antifields are canonically conjugate. At tree level in the quantum theory, the original classical action is extended to a new action S_0 defined on the whole content of $\mathcal{F}$ and exhibiting an off-shell odd symmetry corresponding to the gauge symmetry of the original field theory. The gauge fixing is carried out by restricting the action S_0 to a suitable Lagrangian submanifold $\mathcal{L}$ in $\mathcal{F}$. Gauge independence, that is independence from the choice of $\mathcal{L}$, is ensured if S_0 satisfies the classical BV master equation

$$\{S_0, S_0\} = 0. \tag{1.1}$$

At loop level, quantum corrections modify the action S_0 and turn it into a quantum action $S_\hbar$. Gauge independence is then ensured provided $S_\hbar$ satisfies the quantum BV master equation

$$\hbar\Delta S_\hbar + \frac{1}{2}\{S_\hbar, S_\hbar\} = 0, \tag{1.2}$$

where Δ is a suitably regularized odd functional Laplacian in $\mathcal{F}$. Violations of this correspond to gauge anomalies.

The observables of the field theory constructed in this way are characterized by having gauge independent correlators. The gauge independence of a correlator $\langle\psi_\hbar\rangle$ is ensured if $\psi_\hbar$ satisfies the equation

$$\delta_\hbar\psi_\hbar := \hbar\Delta\psi_\hbar + \{S_\hbar, \psi_\hbar\} = 0. \tag{1.3}$$

The solutions $\psi_\hbar$ of (1.3) are called quantum BV observables. The quantum BV operator $\delta_\hbar$ is nilpotent. Therefore, there is a cohomology associated with it, the quantum BV cohomology. Since correlators of BV exact observables vanish, effectively distinct BV observables are in one-to-one correspondence with the BV cohomology classes.

After this very brief review of BV theory, let us come to the topic of the paper. The algebraic structure consisting of the graded algebra of functionals on the field/antifield space $\mathcal{F}$, the antibracket $\{\cdot,\cdot\}$ and the odd Laplacian Δ is called a BV algebra. It provides the formal framework within which the BV quantization algorithm is implemented. This has motivated a number of mathematical studies of BV algebras [5–7].

The classical field theory originally considered, even if it is a gauge theory, may still have global symmetries. In certain cases, one may wish to gauge these latter. In a BV framework, the gauging of a global symmetry consists in the coupling of the ungauged "matter" field theory and a suitable pure "ghost" field theory corresponding to the symmetry. (Ordinary ghost and gauge fields normally combine in ghost superfields.) Two procedures of concretely working this out are possible in principle.

(i) One couples the matter and the ghost field theories at the classical level, by adding suitable interaction terms, obtaining a gauged classical field theory. Then, one quantizes this latter using the BV algorithm, by constructing the appropriate BV algebra and quantum BV master action.
(ii) One separately quantizes the matter and the ghost field theories, by constructing the appropriate BV algebra and quantum BV master action of each of them. Then, one embeds the matter and ghost BV algebra structures so obtained in a minimal gauged BV algebra structure and constructs a gauged quantum BV master action by adding the matter and ghost actions and suitable interaction terms in a way consistent with the quantum BV master equation.

We call these two approaches *classical gauging* and *BV algebra gauging*, respectively. Superficially, it may look like classical gauging is more natural: after all, BV theory was devised precisely to quantize classical gauge theories. In fact, in certain cases, BV gauging is more advantageous.

In a prototypical example, one efficient way of generating a sigma model on a non-trivial manifold X is the gauging of a sigma model on a simpler manifold Y carrying the action of a Lie group G such that $X \simeq Y/G$ [8,9]. The target space of the gauged model turns out to be precisely X. In a BV formulation of the ungauged sigma model, G acts as a group of global symmetries. The gauging of these is performed by coupling the ungauged model to a suitable ghost sigma model, yielding in a natural way a BV formulation of the gauged model [10–12].

The Alexandrov–Kontsevich–Schwartz–Zaboronsky (AKSZ) formalism of Ref. [13] is a method of constructing solutions of the classical BV master equation directly, without starting from a classical action with a set of symmetries, as is originally done in the BV framework. When building models with gauged global symmetries in a AKSZ framework, BV algebra gauging is definitely more natural and transparent than classical gauging.

In this paper, we study in great detail the BV algebra gauging of a matter field theory with global symmetries. For a certain global symmetry, the ghost field theory to be coupled to the matter theory may have a varying amount of "ghostly supersymmetry". Coupling, if feasible, to an $N = 0$ ghost system yields an ordinary gauge field theory. Coupling to an $N = 1$ ghost system leads to a topological gauge field theory. Coupling to higher N ghost systems yields topological gauge field theories with higher topological supersymmetry. In the latter case, however, problems of a completely new kind show up, which may require a major revision of the standard BV algebraic framework.

Though BV algebra gauging is ultimately carried out within the framework of BV theory, ordinary BV cohomology is not adequate for the classification of observables of the field theories constructed in this way. If $\mathfrak{g}$ is the global symmetry Lie algebra, $\mathfrak{g}$-invariant BV cohomology in the $N = 0$ case, $\mathfrak{g}$-equivariant BV cohomology in the $N = 1$ case and presumably some higher $\mathfrak{g}$-equivariant BV cohomologies for larger N are required.

We shall carry out our analysis of BV gauging in a finite dimensional setting as in [5–7]. This has its advantages and disadvantages. It allows one to focus on the essential features of gauging, especially those of an algebraic and geometric