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The soliton solutions, dromions of the Kadomtsev–Petviashvili and Jimbo–Miwa equations in (3 + 1)-dimensions

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Abstract

By applying the Painlevé test, the Kadomtsev–Petviashvili equation and Jimbo–Miwa equation in (3 + 1)-dimensions are shown to be non-integrable. Through the obtained truncated Painlevé expansions, two bilinear equations are constructed. In addition, starting from the bilinear equations, one soliton, two soliton and dromion solutions are also derived. The analysis of the dromions shows that the interactions of the dromions for the (3 + 1)-dimensional equations may be elastic or inelastic.

1. Introduction

Since the soliton phenomena were first by Scott Russell in 1834 and the Korteweg-de Veries equation was solved by the inverse scattering method by Gardner et al. in 1967, the study of solitons and the related issue of the construction of solutions to a wide class of nonlinear equations has become one of the most exciting and extremely active areas of research investigation [1–10]. Early in the study of soliton theory, the main interests of scientists were restricted to the (1 + 1)-dimensional cases because of the difficulty of finding the physically significant higher-dimensional solutions which are localized in all directions. Recently, the study of the exponentially localized soliton solutions, called dromions, in higher dimensions has attracted much more attention. In particular, some (2 + 1)-dimensional integrable models such as the Davey–Stewartson (DS), Kadomtsev–Petviashvili (KP), Nizhnik–Novikov–Veselov (NNV), dispersive long wave (DLW) equations have been studied by many authors [11–19]. However, to our knowledge, very little research has done on the (3 + 1)-dimensional nonlinear models. It is well known that most of (3 + 1)-dimensional nonlinear models fail the conventional integrability tests, the natural and important problem is that are there dromions for (3 + 1)-dimensional nonlinear models. If exists, are the interactions of dromions elastic or inelastic?

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In this paper, we will consider two nonlinear models in (3 + 1)-dimensions. One of them is the (3 + 1)-dimensional KP equation [20–22]:

$$(u_t + 6uu_x + u_{xxx})_x - 3u_{yy} - 3u_{zz} = 0, \tag{1}$$

which explains soliton wave solution propagation in the field of plasma physics, fluid dynamics etc. Another is the (3 + 1)-dimensional equation of the form [23]:

$$u_{xxxy} + 3u_x u_{xy} + 3u_y u_{xx} + 2u_{yt} - 3u_{xz} = 0, \tag{2}$$

which comes from the second member of a KP hierarchy called Jimbo–Miwa equation. It will be shown that these two models are not Painlevé integrable, but the truncated Painlevé expansions can transform the nonlinear models into the bilinear equations. Starting from the bilinear equations, one soliton solution, two soliton solution and dromions are obtained.

2. The (3 + 1)-dimensional KP equation

2.1. The Painlevé test and bilinear form

There are many methods to check whether nonlinear partial differential equations pass the Painlevé test, among those, the WTC–Kruskal algorithm can not only simplify the Painlevé test, but also obtain some truncated expansions related to integrability at the same time [24]. Very recently, a *Maple* software package **wkptest** based on the WTC–Kruskal algorithm has been developed by us [25]. This package can carry out the traditional Painlevé test for polynomial partial differential equations automatically. Furthermore, some truncated Painlevé expansions are also obtained whether an equation passes the test or not.

Using **wkptest**, we conclude that Eq. (1) fails the Painlevé test, and the details can be found in Ref. [25]. In addition, **wkptest** outputs the following truncated Painlevé expansion,

$$u = -2\phi_x^2\phi^{-2} + 2\phi_{xx}\phi^{-1} + u_2 = 2(\ln\phi)_{xx} + u_2. \tag{3}$$

To find the bilinear form, it is assumed that $u_2 = 0$. Using Eq. (3), the bilinear form of the (3 + 1)-dimensional KP equation (1) is found to be

$$(D_xD_t + D_x^4 - 3D_y^2 - 3D_z^2)\phi\cdot\phi = 0, \tag{4}$$

where the bilinear operator "D" is defined as

$$D_x^mD_y^nD_z^rD_t^s = \left(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'}\right)^m\left(\frac{\partial}{\partial y}-\frac{\partial}{\partial y'}\right)^n\left(\frac{\partial}{\partial z}-\frac{\partial}{\partial z'}\right)^r\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial t'}\right)^s\Big/ x'=x, y'=y, z'=z, t'=t.$$

2.2. Soliton solutions and dromions

Substituting

$$\phi = 1 + \phi^{(1)}\epsilon + \phi^{(2)}\epsilon^2 + \phi^{(3)}\epsilon^3 + \cdots,$$

into (4) and comparing coefficients of same powers of ϵ gives the following recursion relations for the $\phi^{(n)}$

$$\begin{aligned}
&\phi^{(1)}_{xt} + \phi^{(1)}_{xxxx} - 3\phi^{(1)}_{yy} - 3\phi^{(1)}_{zz} = 0,\\
&2(\phi^{(2)}_{xt} + \phi^{(2)}_{xxxx} - 3\phi^{(2)}_{yy} - 3\phi^{(2)}_{zz}) = -(D_xD_t + D_x^4 - 3D_y^2 - 3D_z^2)\phi^{(1)}\cdot\phi^{(1)},\\
&\phi^{(3)}_{xt} + \phi^{(3)}_{xxxx} - 3\phi^{(3)}_{yy} - 3\phi^{(3)}_{zz} = -(D_xD_t + D_x^4 - 3D_y^2 - 3D_z^2)\phi^{(1)}\cdot\phi^{(2)},\ldots
\end{aligned} \tag{5}$$

It is easy to verify that Eq. (4) and (5) have the following solution:

$$\phi = 1 + \exp(\eta), \quad \eta = kx + py + qz + wt, \tag{6}$$

with the dispersion relation $kw + k^4 - 3p^2 - 3q^2 = 0$. According to (3) and (6), the one soliton solution is then obtained

$$u = \frac{k^2}{2}\operatorname{sech}^2\left(\frac{-k^2x - pky - qkz + k^4t - 3p^2t - 3q^2t}{2k}\right).$$