Available online at www.sciencedirect.com
SCIENCE DIRECT®
Chaos, Solitons and Fractals 30 (2006) 83–93
CHAOS SOLITONS & FRACTALS
www.elsevier.com/locate/chaos

Boundedness and global stability for nonautonomous recurrent neural networks with distributed delays ☆

Haijun Jiang *, Zhidong Teng

College of Mathematics and System Sciences, Xinjiang University, Urumqi 830046, China

Accepted 24 August 2005

Abstract

In this paper, we investigate the nonautonomous recurrent neural networks systems with distributed delay. By applying Lyapunov functional method and using some analytic technique, several sufficient conditions to ensure the ultimate boundedness, global asymptotic stability and global exponential stability are established. The results obtained in this paper are new and useful.

1. Introduction

Recurrently connected neural networks, sometimes called Grossberg–Hopdield neural networks have been widely studied both in theory and application. They have been successfully applied in signal processing, pattern recognition and associative memories, especially in processing static images. In recent years, the dynamics of autonomous recurrent neural networks with finite delays have been extensively studied. The basic and important subjects for these systems are the existence of global asymptotic stability, global exponential stability of equilibrium point, and the existence and global stability of periodic solutions. Many important results can be found in [1–15] and references therein.

Up to now, most research on delayed neural networks has been restricted to the cases of constant delays or time-varying delays. Though delays arise frequently in practical applications, it is difficult to measure them precisely. In certain situations, delays are unbounded. That is, the entire history affects the present. Such delay terms, more suitable to practical neural nets. However, a few papers have considered autonomous recurrent neural network with infinite delay. Particularly, in [16–25], the authors established the sufficient conditions to ensure the existence, global asymptotical stability and global exponential stability of equilibrium point for autonomous neural networks with infinite distributed delay. In [26], by constructing proper Lyapunov function and using some analytic technique, the authors obtain several sufficient conditions to ensure the dissipativity of neural networks with both variable and unbounded delays.

However, as we know well, the nonautonomous phenomenon often occurs in many realistic systems. Particularly, when we consider a long-term dynamics of a system, the parameters of the system usually will change along with time.

☆ Supported by The National Natural Science Foundation of PR China (10361004), The Major Project of The Ministry of Education and The Natural Science Foundation of Xinjiang University.
* Corresponding author. Tel.: +86 991 8580535.
E-mail address: jianghai@xju.edu.cn (H. Jiang).

0960-0779/$ - see front matter © 2005 Elsevier Ltd. All rights reserved.
doi:10.1016/j.chaos.2005.08.132

In addition, in many applications, the property of periodic oscillatory solutions of neural networks is also very interesting. For example, the human brain has been in periodic oscillatory or chaos state, hence it is of prime importance to study stability, periodic oscillation, bifurcation and chaos phenomenon of neural networks. Therefore, the research on the neural networks with variable coefficients and infinite delay also is very important and significant like on the autonomous neural networks.

We see that there have been considerable research on the nonautonomous neural networks (for example [27–31]). Particularly, in [27,28,31], the periodic Hopfield neural networks, cellular neural networks and BAM neural networks with variable coefficients and finite delays have been studied. By using the continuation theorem and Lyapunov functional method, the authors established some sufficient conditions to ensure the existence, uniqueness and global stability of periodic solutions. In [29,30], for a class of cellular neural networks with variable coefficients and time-varying delays, under the assumption that the nonlinear response functions may be unbounded, by using Lyapunov functional method and the technique of inequality analysis, the authors established a series of criteria on the boundedness, global exponential stability and the existence of periodic solution and its global exponential stability.

In this paper, we will consider the following nonautonomous recurrent neural networks with infinite distributed delays

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = -c_i(t)x_i(t) + \sum_{l=1}^{m}\sum_{j=1}^{n} a_{ijl}(t) f_{ijl}\left(\int_{-\infty}^{t} k_{ijl}(t-s)x_j(s)\mathrm{d}s\right) + I_i(t), \quad i = 1, 2, \ldots, n, \tag{1}$$

where $t \in R_+ = [0, \infty)$ and $x(t) = (x_1(t), x_2(t), \ldots, x_n(t)) \in R^n$. We assume that functions $c_i(t)$, $a_{ijl}(t)$ and $I_i(t)$ $(i,j = 1,2,\ldots,n,\ l = 1,2,\ldots,m)$ are bounded and continuous defined on $t \in R_+$ and functions $k_{ijl}(s)$: $R_+ \to R_+$ $(i,j = 1, 2,\ldots,n,\ l = 1,2,\ldots,m)$ are integrable.

From the mathematical point of view, systems with constant coefficients and finite delays are different from those with variable coefficients and infinite distributed delays, and known mathematical methods do not directly apply. Here, we develop some methods from functional differential equations to study the stability of nonautonomous recurrent neural networks with infinite distributed delays.

Our main purpose in this paper is to study the ultimate boundedness, global asymptotic stability and global exponential stability of system (1). We will not require that all nonlinear response functions $f_{ijl}(u)$ in system (1) are bounded on R_+. In addition, we also will not require that system (1) has any equilibrium point. We will establish a series of new criteria on the ultimate boundedness, global asymptotic stability and global exponential stability for system (1). The method used in this paper is to construct the Lyapunov function and combine with the boundedness and stability theorems of functional differential equation [32,33]. In addition, we introduce many parameters and utilize the techniques of inequality analysis. We will see that the results obtained in this paper are new and useful.

The organization of this paper is as follows. In Section 2, we will give some preliminaries. In Sections 3–5, we will establish respectively our main results on the boundedness, global asymptotic stability, global exponential stability for system (1), respectively. An example is also given in Section 6 to illustrate the main results of this paper. In Section 7, we will give some concluding remarks.

2. Definitions, assumptions and lemmas

We denoted by C the Banach space of continuous functions $\phi(\theta) = (\phi_1(\theta), \phi_2(\theta), \ldots, \phi_n(\theta)) : (-\infty, 0] \to R^n$ with the norm

$$\|\phi\| = \sup\nolimits_{-\infty<\theta\leqslant 0}|\phi(\theta)|, \quad \text{where } |\phi(\theta)| = \left[\sum_{i=1}^{n}\phi_i^2(\theta)\right]^{1/2}.$$

In this paper, we choose that C is the phase space of system (1). For any $t_0 \in R_+$ and $\phi = (\phi_1, \phi_2, \ldots, \phi_n) \in C$ we denote by $x(t, t_0, \phi)$ the solution of system (1) satisfying the following initial condition

$$x_{t_0}(\theta) = x(t_0 + \theta) = \phi(\theta) \quad \text{for all } \theta \in (-\infty, 0]. \tag{2}$$

On the boundedness and global stability for system (1), we have the following definitions.

Definition 1. System (1) is said to be uniformly bounded, if for any constant $\alpha > 0$ there is a constant $B = B(\alpha) > 0$ such that for any $t_0 \in R_+$ and $\phi \in C[-\tau, 0]$ with $\|\phi\| \leqslant \alpha$, one has

$$|x(t, t_0, \phi)| \leqslant B \quad \text{for all } t \geqslant t_0.$$