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Exponential stability of neural networks with variable delays via LMI approach

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Abstract

This paper presents sufficient conditions for global asymptotic/exponential stability of neural networks with time-varying delays. By using appropriate Lyapunov–Krasovskii functionals, we derive stability conditions in terms of linear matrix inequalities (LMIs). This is convenient for numerically checking the system stability using the powerful MATLAB LMI Toolbox. Compared with some earlier work, our result does not require any restriction on the derivative of the delay function. Numerical example shows the efficiency and less conservatism of the present result.

1. Introduction

Time delays inevitably exist in neural networks due to various reasons. For example, time delays can be caused by the finite switching speed of amplifier circuits in neural networks [1] or deliberately introduced to achieve tasks of dealing with motion-related problems, such as moving image processing [2]. The existence of time delays may degrade system performance and cause oscillation in a network, leading to instability. Study of time delay effects on stability and convergent dynamics of neural networks has received considerable attention in the past decades (see, e.g., [1–19] and the references therein).

While many works considered the case of neural networks with constant time delays [1–12], there are also practical cases where time delays are uncertain and may be time varying. Recently, there has been an increasing interest in addressing stability of neural networks with time varying delays (e.g., [13–19]). It is often assumed in these studies that the time delay function is continuously differentiable and its derivative does not exceed the unity [15–19]. This is a very restrictive condition due to the use of some specific Lyapunov–Krasovskii functionals in deriving the stability conditions.

In this paper we will employ a new Lyapunov–Krasovskii functional for establishing exponential stability conditions for neural networks with time-varying delays. Our conditions do not impose any restriction on the derivative of time

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delay functions and are expressed in terms of linear matrix inequalities (LMIs), which can be checked numerically using the effective LMI toolbox in MATLAB. Compared with some existing results on neural networks with constant time delays, the presented conditions can give greater delay bound for stability, leading to less conservative conditions.

This paper is organized as follows. Section 2 gives the model description and some preliminary results. Section 3 presents the main result of this paper on the LMI stability conditions. In Section 4, an example is given to show the effectiveness of the obtained results. Finally, a brief conclusion is drawn in Section 5.

2. Model and preliminaries

The neural network with time varying delay considered in this paper is described by the following differential equation:

$$\dot{u}(t) = -Cu(t) + Ag(u(t)) + Bg(u(t-\tau(t))) + I \tag{1}$$

where $u(t) = [u_1(t), u_2(t), \ldots, u_n(t)]^{\mathrm{T}}$ is the neuron state vector, $C = \mathrm{diag}(c_1, \ldots, c_n) > 0$ is the relaxation matrix, $A = (a_{ij})_{n\times n}$ and $B = (b_{ij})_{n\times n}$ are weight matrices, $g(u) = [g_1(u_1), g_2(u_2), \ldots, g_n(u_n)]^{\mathrm{T}}$ denotes the neuron activations, $I = [I_1, I_2, \ldots, I_n]^{\mathrm{T}}$ is the constant external input vector, and $\tau(t)$ is a continuous function describing the time-varying transmission delays in the network system and satisfies $0 \leqslant \tau(t) \leqslant h$ for all $t \geqslant 0$, with h a constant.

Throughout this paper we assume $g_j(u_j)$ are bounded and satisfy the following condition:

$$|g_j(x) - g_j(y)| \leqslant l_j|x - y| \tag{2}$$

where $l_j > 0$ are constants for $j = 1, 2, \ldots, n$.

With the boundedness of function g_i, it follows readily from Brouwer's fixed point theorem that for a given constant input vector I, system (1) has at least one equilibrium point $u^* = [u_1^*, u_2^*, \ldots, u_n^*]^{\mathrm{T}}$ determined by

$$Cu^* = (A + B)g(u^*) + I$$

For convenience in discussion, we shift u^* to the origin by taking the transformation $x = u - u^*$ and write Eq. (1) into the form

$$\dot{x}(t) = -Cx(t) + Af(x(t)) + Bf(x(t-\tau(t))) \tag{3}$$

where $x = [x_1, x_2, \ldots, x_n]^{\mathrm{T}}$, $f(x) = [f_1(x_1), f_2(x_2), \ldots, f_n(x_n)]^{\mathrm{T}}$ with $f_j(x_j) = g_j(x_j + u_j^*) - g_j(u_j^*)$. By condition (2), $f_j(x_j)$ satisfy

$$|f_j(x_j)| \leqslant l_j|x_j|, \quad j = 1, 2, \ldots, n \tag{4}$$

The solution of system (3) is dependent on an initial condition $x(\theta)$ for $-h \leqslant \theta \leqslant 0$.

In the following, we will use the notation $X > 0$ ($X < 0$) to denote a symmetric and positive definite (negative definite). To obtain our stability conditions, we need the following technical results.

Lemma 1 [20]. *For any vectors $a, b \in \mathbb{R}^n$ and any positive definite matrix $Y \in \mathbb{R}^{n\times n}$, the following inequality holds*:

$$2a^{\mathrm{T}}b \leqslant a^{\mathrm{T}}Ya + b^{\mathrm{T}}Y^{-1}b$$

Lemma 2 [20]. *For a symmetric matrix*

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^{\mathrm{T}} & S_{22} \end{bmatrix}$$

the following conditions are equivalent:

(i) $S < 0$.
(ii) $S_{11} < 0$ *and* $S_{22} - S_{12}^{\mathrm{T}}S_{11}^{-1}S_{12} < 0$.
(iii) $S_{22} < 0$ *and* $S_{11} - S_{12}S_{22}^{-1}S_{12}^{\mathrm{T}} < 0$.

3. Stability analysis

First we consider the global asymptotic stability of system (3).