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Chaos, Solitons and Fractals 29 (2006) 307–312
CHAOS SOLITONS & FRACTALS
www.elsevier.com/locate/chaos

Compact-like kink in a real electrical reaction–diffusion chain

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Accepted 8 August 2005

Abstract

We demonstrate experimentally the compact-like kinks existence in a real electrical reaction–diffusion chain. Our measures show that such entities are strictly localized and consequently present a finite spatial extent. We show equally that the kink velocity is threshold-dependent. A theoretical quantification of the critical coupling under which propagation fails is also achieved and reveals that nonlinear coupling leads to a propagation failure reduction.

1. Introduction

Reaction–diffusion equations arise in many fields of biology, ecology, chemistry and physics [1]. For instance, they are used to describe the dispersive behavior of cell or animal populations as well as chemical concentrations. Indeed, for a single species in three space dimensions, the general conservation of particle density v leads to an equation of the form,

$$\frac{\partial v}{\partial t} + \nabla F = f(v), \tag{1}$$

where, F is a general *flux transport* owing to diffusion or some other processes, and $f(v)$ is a nonlinear continuous function describing the rate of particle creation or a source reaction term. For the specific case, where $F = \kappa \nabla v$, and κ is a constant corresponding to a diffusion coefficient, Eq. (1) in one dimension reduces to,

$$\frac{\partial v}{\partial t} = \kappa \frac{\partial^2 v}{\partial x^2} + f(v). \tag{2}$$

Eq. (2) is also found in cardiophysiology, or neurophysiology to describe the wave propagation in nerve cells, where v represents the membrane potential, and $f(v)$ models the ionic channel dynamics that is the ionic flux between the extracellular and intracellular media. Nowadays, the qualitative behavior of such scalar reaction–diffusion equations in one dimension is relatively well understood [2,3], which is not yet the case in more than one dimension, coupled reaction–diffusion equations systems and spatial discrete systems. Since recent years, it has become clear that continuous

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0960-0779/$ - see front matter
doi:10.1016/j.chaos.2005.08.212

reaction–diffusion equations of the general form (2) provide an inadequate description of weakly coupled systems, where the interplay between nonlinearity and discreteness can lead to novel effects not present in the continuum models. Indeed, front waves propagation failure in discrete excitable media, is certainly the most important example, and is easily observed in the discrete version of Eq. (2)

$$\frac{dv_n}{dt} = K(v_{n+1} - 2v_n + v_{n-1}) + f(v_n), \tag{3}$$

when $f(v_n)$ is a bistable function presenting the form $f(v_n) = (v_n^2 - 1)(v_n - \alpha)$, homogeneous to a force deriving from a free energy, with $-1 < \alpha < +1$, as presented in Fig. 1. Mathematical or more precisely numerical solutions of such equations are very extent in space and/or time when the coupling term is not very small. However, patterns observed in nature either stationary or traveling present a finite extent. Indeed, recently it has been shown by Rosenau and Hyman [4–6], that solitary-wave solutions may be compactified under the influent of nonlinear dispersion which is able to cause deep qualitative changes in the nature of nonlinear phenomena. Such robust soliton-like solutions characterized by the absence of infinite tails or wings and whose width is velocity independent, have been called compatons [5–8]. Since few time, it has been equally shown that discrete reaction–diffusion equations with a nonlinear coupling admit exact compact-like kink or pulse solutions [9]. Thus, the purpose of the present paper is to show experimentally the theoretical prediction of [9] and consequently demonstrate that the concept of compactification in nonlinear diffusive media presenting a nonlinear coupling is a reality, and that the main characteristics as linear coupled systems are conserved. Such models could find their place in neuronal context for instance as advanced nerves model. Indeed, this model takes into account the real finite extent (in space and time) of action potential or senile plaques in cerebral cortices regarding Alzheimer's disease [10], and consequently may give an opening explanation to not understood phenomena. The outline of this paper is as follow: First, we recall the analytical results and present our experimental device. Then, in Section 2 we show the compact-like kink propagation and the temporal dynamics of ones propagation of one cell experiencing the wave front passage. Section 3, we present and discuss the wave front velocity versus the threshold parameter α, and compare it to numerical simulations. A quantification of the propagation failure mechanism is presented in Section 4. Finally, Section 5 is devoted to concluding remarks.

2. Theoretical recall and experimental device introduction

As we said above, the lattice model that we consider is a chain of overdamped oscillators anharmonically coupled to their nearest neighbors and interacting with a nonlinear substrate potential $V(v_n)$. The equation of the system is then given by

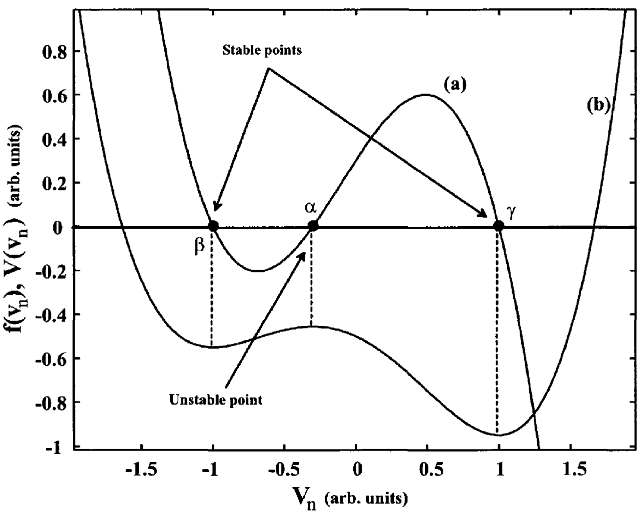

Fig. 1. (a) Nonlinear bistable function $f(v_n)$, deriving from the substrate potential $V(v_n)$. (b) α corresponds to an unstable point homogeneous to a threshold, while β and γ correspond to stable points.