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Cantorian space–time and Hilbert space: Part II—Relevant consequences

G. Iovane

Dipartimento di Ingegneria dell'Informazione e Matematica Applicata, Universitá di Salerno, Italy

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Abstract

In this paper, we will show the consequences of the link between $\varepsilon^{(\infty)}$ and $H^{(\infty)}$. Starting from El Naschie's $\varepsilon^{(\infty)}$ nature shows itself as an arena where the laws of physics appear at each scale in a self–similar way, linked to the resolution of the observations; while Hilbert's space $H^{(\infty)}$ is the mathematical support to describe the interaction between the observer and dynamical systems.

The present formulation of space–time, based on the non-classical, Cantorian geometry and topology of the space–time, automatically solves the paradoxical outcome of the two-slit experiment and duality. The experimental fact that a wave-particle duality exists is an indirect confirmation of the existence of $\varepsilon^{(\infty)}$. Another direct consequence of the fact that real space–time is the infinite dimensional hierarchical $\varepsilon^{(\infty)}$ is the existence of the scaling law $R(N)$. The present author proposed it as a generalization of the Compton wavelength. This rule gives an answer to segregation of matter at different scales; it shows the role of fundamental constants like the speed of light and Plank's constant h in the fundamental lengths scale without invoking the methodology of quantum mechanics.

In addition, we consider the genesis of *E*-Infinity. A Cantorian potential theory can be formulated to take into account the geometry and topology of $\varepsilon^{(\infty)}$ in the context of gravitational theories. Consequently, we arrive at the result of the existence of gravitational channels.

1. Introduction

This paper is the companion of an earlier one [1]. Reading carefully El Naschie's papers and the previous contribution, it appears clearly that *E*-Infinity theory is a new framework for understanding and describing nature. Probably the main point of the theory is the fact that everything we see or measure is resolution dependent. As reported by El Naschie, in his *E*-Infinity view, space–time is an infinite dimensional fractal that happens to have $D = 4$ as the expectation value for the topological dimension [2]. In detail, the topological dimension $3 + 1 = 4$ means that in our low energy resolution, the world appears to us as if it were four-dimensional. As showed by the author in [3], the observations of the large scale structures show that the dimension changes if we consider different energies, corresponding to different

E-mail address: iovane@diima.unisa.it

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scale lengths in the universe. Therefore, the dimension becomes resolution dependent; consequently it all depends on the energy scale with which we are making our observation.

The vision presented by El Naschie for the micro-world and by the present author for the macro-physics, suggests a radical change based on Cantorian space–time. Here $\epsilon^{(\infty)}$ Cantorian space–time is the physical space–time, where nature manifests its transfiniteness; while as we have seen in [1] and according to the results presented in [4,5] Hilbert's space $H^{(\infty)}$ is a mathematical framework to describe the interaction between the observer and the dynamical system under measurement.

The present formulation, based on the non-classical Cantorian geometry and topology of space–time, automatically solves the paradoxical outcome of the two-slit experiment. As we will see in detail, the measurement, from a mathematical point of view, is equivalent to a projection of $\epsilon^{(\infty)}$ on $H^{(\infty)}$ based on a $3+1$ Euclidean space. As predicted in some papers by El Naschie, the mathematical solution of the two-slit experiment is the physical realization of Gödel's indecidability.

If the present interpretation is correct, then difference from micro- to macro-physics only depends on the resolution by which the observers look at the world. The confirmation of this relevant question is given in [3], where it is pointed out that nature shows us structures with scaling rules, where clustering properties from cosmological to nuclear objects reveal a form of hierarchy. Moreover, the author considered the compatibility of a stochastic self-similar, fractal universe with the observation and the consequences of this model. In detail, it was demonstrated that the observed segregated Universe is the result of a fundamental self-similar law, which generalizes the Compton wavelength relation, $R(N) = (h/Mc)N^{\phi}$, where R is the radius of the structures, h is the Planck constant, M is the total Mass of the self-gravitating system, c the speed of light, N the number of nucleons within the structures, and $\phi = \frac{\sqrt{5}-1}{2}$ is the Golden Mean. It appears that the Universe has a memory of its quantum origin as suggested by Sir Roger Penrose with respect to quasi-crystal [6]. Particularly, the model is related to Penrose tiling and thus to $\varepsilon^{(\infty)}$ theory (Cantorian space–time theory) as proposed by El Naschie [7,8] as well as with connes non-commutative geometry [9]. In [10] waveguiding and mirroring effects were considered with respect to the large scale structure of the Universe. Here, we show that this mechanism could be more general than in the previous paper and deeply linked with the geometry and topology of space–time.

The paper is organized as follows: Section 2 specializes some well known results in functional analysis to the Cantorian space $\varepsilon^{(\infty)}$; Section 3 is devoted to study a Cantorian potential theory and the dependence on the resolution; with Section 4 we consider the genesis of $\varepsilon^{(\infty)}$; while in Section 5 we study the waveguide channels in $\varepsilon^{(\infty)}$ and mirroring effects; finally conclusions are drawn in Section 6.

2. Functional analysis and its applications to $\varepsilon^{(\infty)}$

The well known results, in functional analysis and presented hereunder, are useful for our purpose (for more details see [1,11,12]); here we summarize only some results to show the link between E-Infinity and Hilbert and Sobolev spaces.

Let Ω be a non-empty open set in $\mathbb{R}^m$.

Definition 2.1. Let us consider $p \in \mathbb{R}$ with $1 \leqslant p < \infty$, we pose

$$L^p(\Omega,\mu) = \{f : \Omega \to \mathbb{R},\ \text{with } f \text{ measurable and } |f|^p \in L^1(\Omega,\mu)\}, \tag{2.1}$$

with

$$\|f\|_{L^p} = \left(\int_\Omega |f|^p \,\mathrm{d}\mu\right)^{1/p}. \tag{2.2}$$

Moreover, we pose

$$L^\infty(\Omega,\mu) = \{f : \Omega \to \mathbb{R}\ | f \text{ is measurable and } \exists c \in \mathbb{R} : |f(x)| \leqslant c \text{ almost everywhere in } \Omega\} \tag{2.3}$$

with

$$\|f\|_{L^\infty} = \inf\{c; ||f(x)| \leqslant c \text{ almost everywhere in } \Omega\}. \tag{2.4}$$

Definition 2.2. Let us consider a finite or infinite complete vector space H on the field of complex numbers. In this space, a scalar product is defined so that for $\psi(x),\ \varphi(x) \in H$: