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Fractal black holes and information

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Abstract

If nature is fractal as it evidently is, at classical resolution and if it is suspected to also be fractal at the quantum resolution as it is now a days generally believed to be, then we must have over looked at least two points or so in our physical model building of mini black holes. To start with at such ultra high resolution, the mini black hole geometry must be a fractal. Consequently we have zero volume and only a fractal surface area. Second because we cannot take the differential limit for the ℓ_p^2 covering the transfinite surface area, there will be many gaps between the $(\ell_p)^2$ tilings. In other words we must introduce transfinite corrections to the final result. Proceeding this way the information entropy unit of a black hole should be

$$a = I = (7 + \phi^3)(10)^{-66}\ \text{cm}^2 = 7.23606799(10)^{-66}\ \text{cm}^2$$

The nearest classical result to the above is that obtained by Gerard 't Hooft

$$a = I = (0.724)(10)^{-65}\ \text{cm}^2$$

The paper ends with a general discussion of *E*-infinity theory and its possible relation with 't Hooft's holographic principle and his gluons–quark strings.

1. Introduction

At least for the last two decades there has been a surge in the number of research papers on various types of black holes [1]. This is partly due to the great theoretical importance of the subject, but also in no minor measure because of various misconceptions which were unwittingly injected into the debate on black holes [2–5]. The present paper will focus on two possible aspects which we feel may have been overlooked and may equally help in demystifying certain aspects of black hole physics. More precisely, black holes are envisaged as more or less smooth spheres. However if our basic assumptions about fuzziness of topology and fractalness of the geometry of nature is correct [6–8], then black holes at the Planck scale could not possibly be smooth. This and related questions will form the basis of the present discussion which will highlight the importance of random fractal geometry in high energy physics.

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2. The Bekenstein–Hawking entropy

With the provision that it is a totally unexpected departure from earlier work, the Bekenstein–Hawking entropy is usually derived starting from the formula of Hawking temperature [9]

$$T_{\mathrm{H}} = \frac{1}{8\pi MG}$$

where M is the mass of the black hole and G is Newton's constant. Setting M to mean energy and T_{H} to mean temperature, then entropy S may be found from the well known thermo-dynamical relation

$$T = \frac{\mathrm{d}E}{\mathrm{d}s} = \frac{\mathrm{d}M}{\mathrm{d}s}$$

Inserting one finds

$$\frac{1}{8\pi MG} = \frac{\mathrm{d}M}{\mathrm{d}s}$$

That means

$$\int \mathrm{d}s = \int (8\pi G)M\,\mathrm{d}M = (8\pi G)\int M\,\mathrm{d}M$$

Thus

$$S = (8\pi G)\frac{1}{2}M^2 = 4\pi GM^2$$

where we have ignored the constant of the undetermined integration.

Recalling that the Schwarzschild radius of the black hole is given by [1,2]

$$R_{\mathrm{sch}} = 2MG$$

and therefore the surface area of the black hole horizon is given by

$$4\pi R_{\mathrm{sch}}^2 = 4\pi(4M^2G^2)$$

then the black hole entropy can be written as

$$S = \frac{\text{Area}}{4G}$$

The most remarkable feature of this formula is commonly accepted to be the fact that S depends on the surface area rather than the volume of the black hole. This is reminiscent of the elementary physical fact that the total charge of an electrically charged solid sphere of metal is related to the surface area, not the volume. It is now possible to calculate approximately the number of states associated with S by writing [10]

$$N = \mathrm{e}^S = \mathrm{e}^{(\text{Area}/4G)}$$

Our next step is to divide the surface of the black hole horizon into units constituting quantum bits. In effect, we have introduced a binary system of zero and one as shown in Fig. 1. Therefore it is useful to write N to the base of 2 rather than e. This leads us to

$$N = 2^{A/a}$$

where A is the total horizon area of the black hole and "a" is our quantum information unit given by

$$a = I = 4\ln 2G$$

Since the Planck length is given by

$$\ell_{\mathrm{p}} = \sqrt{G}$$

and choosing units for which $\hbar = C = 1$ one finds [1]

$$\ell_{\mathrm{p}} = \sqrt{G} = 1.616(10)^{-33}\ \mathrm{cm}$$

The unit area is thus

$$a = (4)(\ln 2)(1.616)^2(10)^{-66}\ \mathrm{cm}^2 = (7.240493454)(10)^{-66}\ \mathrm{cm}^2$$