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Horseshoes in a class of simple circuits

Yan Huang [a,b], Xiao-Song Yang [a,*]

[a] Department of Mathematics, Huazhong University of Science and Technology, Wuhan 430074, PR China
[b] Department of Control Science and Engineering, Huazhong University of Science and Technology, Wuhan 430074, PR China

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Abstract

In this paper we study dynamics of a class of simple single-scroll and two-scrolls type circuits, we show that these circuits are chaotic by giving a rigorous verification for existence of horseshoes in these systems. We prove that the Poincaré map derived from the ordinary differential equations of the single-scroll circuit system is semi-conjugate to the 2-shift map; and the Poincaré map of the two-scrolls circuit system is semi-conjugate to the 4-shift map.

1. Introduction

We study a class of simple circuits described by Sprott [1,2]. These circuits are modeled by simple third-order differential equations whose solutions display a rich variety of chaotic and periodic behavior. Like the well-known Chua's circuits [3], these circuits are well suited for qualitative demonstrations and as serious research tool for studying of synchronization chaos circuits, higher-dimensional chaos, and other topics within nonlinear dynamics.

Sprott and his co-workers have studied the circuits by means of bifurcation and Lyapunov exponents. Although the method of Lyapunov exponents is a powerful tool in studying chaotic behavior of dynamic system, the computation of Lyapunov exponents can only provide evidences of chaos because of limited time of computation. With the growing use of nonlinear analysis techniques in many areas of science, it is becoming increasingly important to provide a rigorous verification of chaos. In this paper we revisit a single-scroll and a two-scrolls circuit systems, and present a computer-assisted proof of existence of horseshoes in these systems, thus showing the existence of chaos in these systems.

The paper is structured as follows. In Section 2, some simple circuits are revisited. In Section 3, a topological horseshoe theorem is reviewed. In Section 4, a rigorous computer-assisted proof is presented. In Section 5, some concluding remarks are offered.

* Corresponding author.
E-mail address: yangxs@cqupt.edu.cn (X.-S. Yang).

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2. Sprott's simple circuits

2.1. Single-scroll circuit

One class of simple circuits by Sprott [2] that lead to chaotic behavior are described by the following third-order differential equation:

$$\dddot{x} = -A * \ddot{x} - \dot{x} + D(x) - a, \tag{1}$$

where x represents the voltage at a particular node in the corresponding circuit. In Eq. (1) A and α are constants, the dots denote derivatives with respect to time, and $D(x)$ is a nonlinear function that characterizes the nonlinearity in the circuit.

In this paper we study a circuit described by Eq. (1). The nonlinearity in the circuit models a function proportional to $\min(x, 0)$. Sprott obtained the following relations among the voltages [2]:

$$\begin{aligned} V_1 &= -RC\frac{\mathrm{d}x}{\mathrm{d}t} = -\dot{x}, \\ V_2 &= -RC\frac{\mathrm{d}V_1}{\mathrm{d}t} = \ddot{x}, \\ V_3 &= -V_1 - D(x), \\ RC\frac{\mathrm{d}V_2}{\mathrm{d}t} &= -\left(\frac{R}{R_v}\right)V_2 - \left(\frac{R}{R_0}\right)V_0 - V_3. \end{aligned} \tag{2}$$

The variable resistor R_v acts as a circuit parameter, moving the system in and out of chaos, and the input voltage V_0 may be either positive or negative. The $D(x)$ represents the nonlinearity in the circuit, which is necessary for the circuit to exhibit chaotic behavior. The circuit contains three successive inverting integrators with outputs V_2, V_1 and x, as well as a summing amplifier with its output at V_3. For details, see [2].

In this paper, we will study a concrete nonlinear circuit of the form Eq. (1). The nonlinearity used here models the function $D(x) = -6\min(x, 0)$ and does not lead to unbounded solutions. The resulting circuit is generally much more convenient to work with as commented by [2], and the equations turn out to be of the following form:

$$\begin{aligned} \dot{x} &= -\frac{y}{RC}, \\ \dot{y} &= -\frac{z}{RC}, \\ \dot{z} &= \left(-\frac{Rz}{R_v} - \frac{RV_0}{R_0} + y + 6 \times \min(x, 0)\right)\Big/(RC). \end{aligned} \tag{3}$$

For the parameters $R = 47$, $R_0 = 157$, $R_v = 72.1$, $V_0 = 0.25$, Eq. (3) has a single-scroll attractor as shown in Fig. 1.

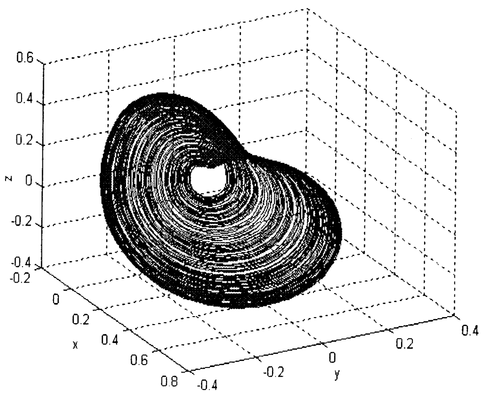

Fig. 1. The attractor of Eq. (3).