Journal of Geometry and Physics 59 (2009) 369–373

Contents lists available at ScienceDirect

Journal of Geometry and Physics

journal homepage: www.elsevier.com/locate/jgp

A higher twist in string theory

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ARTICLE INFO

Article history:
Received 6 June 2007
Received in revised form 10 November 2008
Accepted 20 November 2008
Available online 25 November 2008

Keywords:
Twisted cohomology
K-theory
Heterotic string theory

ABSTRACT

Considering the gauge field and its dual in heterotic string theory as a unified field, we show that the equations of motion at the rational level contain a twisted differential with a novel degree seven twist. This generalizes the usual degree three twist that lifts to twisted K-theory and raises the natural question of whether at the integral level the abelianized gauge fields belong to a generalized cohomology theory. Some remarks on possible such extension are given.

1. Introduction

The classification of the Ramond–Ramond fields in type II string theory has been an important piece of progress in recent years. In the absence of background fields, those are classified by K-theory of spacetime [1,2]. In the presence of the Neveu–Schwarz fields, the RR fields are then described by twisted K-theory. The twisting in type II string theory comes from the NSNS sector. Of particular interest is the twist coming from the rank three field H_3 which shows up in IIA and IIB string theory. One can pass from (twisted) K-theory to (twisted) cohomology through the (twisted) Chern character, which is considered as a map from the former to the latter. In general, what is detected by (twisted) K-theory that is different from that of (twisted) cohomology is the torsion information. At the rational level, the two descriptions coincide, and cohomology is isomorphic to K-theory. Then, on the cohomology side, at the rational level, the fields of classical supergravity satisfy the equations that specify de Rham cohomology, namely $dF = 0$ with the nilpotency condition $d^2 = 0$. Similar description for type I string theory can be given in terms of twisted KO theory (see [3]). What about heterotic string theory?

In the supergravity multiplet of heterotic string theory there is only one potential B_2, whose field strength is H_3. The natural question is whether there is a generalized cohomology description of this in analogy to what happens in type I and type II string theories. Freed [4] classified H_3 and its dual H_7, with potentials B_2 and B_6, via $KOSp$-theory. The charges associated with B_2 lie in $KO^0(X)$, but due to the presence of a magnetic current with a self-duality condition, the field B_2 itself does not belong to $KO^1(X)$ but to the cohomology theory $KOSp^\bullet = KO^\bullet \times KSp^\bullet$.

We know that in heterotic theories there is a coupling between the H-field and the Chern–Simons form of the gauge theory via the Manton–Chapline coupling [5,6]. This suggests that the gauge fields, being related to H that way, might have an interpretation of their own in terms of generalized cohomology. It is the purpose of this note to uncover such a structure. We work at the rational level and then propose the generalized cohomology lift. We are thus looking at a cohomology theory related to the gauge fields, and while Reference [4] looks at Maxwell's system for H_3 and H_7, we consider the system for F_2 and $*F_2$.

Note that a curvature being a K-theory element already appears in type IIA string theory where $F_2 = dA$ (in the constant dilaton case, see [7]) which is the curvature of the spacetime bundle – the M-theory circle bundle – and is interpreted as the

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0393-0440/$ – see front matter
doi:10.1016/j.geomphys.2008.11.009

RR two-form, and fits into the K-theoretic description of the total RR field. We hope that, with this in mind, the transition to considering the gauge fields in heterotic string theory as elements in generalized cohomology should perhaps not sound too conceptually strange.

We assume an abelian reduction of the Yang–Mills group. We define a total gauge field which satisfies a twisted Bianchi identity, except that now the twist is given by the degree seven field H_7. We consider the corresponding complex and the generator leading to a uniform degree differential. The appearance of a higher degree generator, which we identify, makes contact with the discussion on higher generalized cohomology in string theory [8–11] and in M-theory [12–14].

The note is organized as follows. After reviewing the standard three-form twist, given by the NS H-field, on the Ramond–Ramond fields, we present a degree seven twist in heterotic string theory. What is being twisted is not the RR fields, but rather the abelianized *gauge* fields. We show that the twist makes up a differential, i.e. the expression squares to zero. As usual, it is tempting to lift the rational equation to include torsion. We do not have a final answer on which generalized cohomology theory is the right one, but we discuss some possibilities which fit into the discussion of the RR fields for type II string theory [8–11] and for type I [4]. We should point out a caveat. This is that we do not have a clear handle on heterotic string theory like we do on type II string theory. In the latter case, a number of points support the proposal of a relationship between K-theory and string theory. Among the most obvious are the observation about RR charges and K-theory [15] and the relationship between the geometry of K-theory and the geometry of Chan–Paton bundles over D-branes. However, the situation in heterotic string theory is far from being analogous. The arguments provided here are based on duality. The final answer could be arrived at through a derivation from the structure of the theory, or through an example which includes subtle torsion fields. Both are outside the scope of this note.

2. Review of the three-form twist in type II

Here we recall the known case. In string theory, motivated by K-theory, one can combine the Ramond–Ramond of different ranks, into one RR object as

$$F = \sum_i^n F_i, \tag{2.1}$$

where $i = 2p$ for IIA ($i = 2p + 1$ for IIB) and $n = 10$ for IIA (and $n = 9$ for IIB). Here we have a twist by the B-field

$$\mathrm{d}_{H_3} = \mathrm{e}^{B_2}\mathrm{d}\mathrm{e}^{-B_2} = (\mathrm{d} - \mathrm{d}B_2\wedge) = (\mathrm{d} - H_3\wedge). \tag{2.2}$$

This differential satisfies

$$\mathrm{d}_{H_3}^2 = \mathrm{d}^2 + H_3 \wedge \mathrm{d} - H_3 \wedge \mathrm{d} - \mathrm{d}H_3 \wedge - H_3 \wedge H_3\wedge, \tag{2.3}$$

which gives zero because of (2.2), and the fact that the wedge product of two copies of an abelian odd-degree form is zero. Thus it defines a twisted form of de Rham theory $\Omega^k(X)$ but with the differential d_{H_3}. This can be lifted to twisted K-theory. Although the above presentation involved a cohomologically trivial H-field, the result is of course the same for the case $[H_3] \neq 0$. The equation of motion of the RR fields in twisted cohomology is

$$\mathrm{d}F_n - H_3 \wedge F_{n-2} = 0. \tag{2.4}$$

3. The seven-form twist in the heterotic theory

Let us start with a general action of the form

$$S = \int H_3 \wedge *H_3 + \int F_2 \wedge *F_2 \tag{3.1}$$

with a Chapline–Manton coupling $H_3 = CS_3(A)$, where $CS_3(A)$ is the Chern–Simons three-form for the gauge field A, whose curvature is $F_2 = \mathrm{d}_A A$ (d_A is the gauge covariant derivative). This is part of the coupling of type I supergravity to Yang–Mills theory, say in heterotic string theory. We would like to consider the case where the Yang–Mills group, $E_8 \times E_8$ or $\mathrm{Spin}(32)/\mathbb{Z}_2$, is broken down to an abelian subgroup, We assume manifolds M^{10} such that this breaking via Wilson lines is possible. We could consider the Cartan torus for example.

Let us vary the action with respect to A in order to get the gauge field equation of motion. We have (assuming the abelian case)

$$\frac{\delta S}{\delta A} = *H_3 \wedge F_2 - \mathrm{d} * F_2 \tag{3.2}$$

which implies the equation for the gauge field,

$$(\mathrm{d} * - * H_3\wedge)F_2 = 0. \tag{3.3}$$

We manipulate the above equation to put it in a more suggestive form. Namely, in order to write the operator as $(\mathrm{d} - \mathcal{O})$, we define the combined curvature

$$\mathcal{F} = F_2 + *F_2 \tag{3.4}$$