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Chaos, Solitons and Fractals 27 (2006) 868–880
CHAOS
SOLITONS & FRACTALS
www.elsevier.com/locate/chaos

# On the "relativistic" description of motion of soliton-like defects in elastic media

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**Abstract**

An analysis of the manner of establishing a relativistic micro-universe with respect to the motion of soliton-like defects in elastic media is performed. It is demonstrated that the change of variables in the elastic-dynamic equations holding the motion of a screw dislocation must be complemented by the contraction law for the displacement vector and that a theory based on Lorentz's transformations is not the only possible framework for representing the motion of soliton-like defects.


## 1. Introduction

In the framework of classical and continuum mechanics there are several effects analogous to relativistic ones which, for different reasons, have attracted the attention of many researchers until very recently [1,2].

In fact, even if at first sight it may seem that the special theory of relativity does not possess a classical analogy, and certainly does not admit any universal velocity other than that of light, already in 1938 Frenkel and Kontorova [3], with a phenomenological one-dimensional sine-Gordon-like model, and, successively, in 1949 Frank [4], showed that the motion of a screw dislocation in an elastic continuum, which can be regarded as the motion of a topological soliton in a solid, is accompanied by effects very similar to the relativistic ones. Actually, similar to the relativistic particles, these solitons have a continuous and limited range of velocities where the upper bounding role of the velocity of light is played by the transverse acoustic, or shear wave, velocity in the medium, $c_T$. These observations were experimentally confirmed in 1957 by Johnston and Gilman [5] using tests on lithium fluoride crystals, where it has been shown that on the velocity of a dislocation, $v$, approaching $c_T$ the amount of energy required to keep the motion would steeply increase to infinity. In this context we use a broader definition of soliton than usually mathematicians do. Indeed, while mathematicians call soliton a localised particle-like solution of a completely integrable nonlinear equation having finite energy [1,6] here, like most physicists, we say that a soliton is any perturbation of a homogeneous or spatially periodic medium characterized by some kind of stability property. Conversely, these objects do not need to be regarded as entities deriving from quantization of classical fields, like phonons and excitons, even if some of the results presented here could be extended to the behaviour of these particles.

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doi:10.1016/j.chaos.2005.05.032

The principal analogies with the special theory of relativity which can be pointed out with reference to the motion of soliton-like defects in elastic media are the length contraction of a moving soliton and the law which expresses the dependence of the soliton energy on its velocity.

Quite naturally the question arises if these relativistic effects, apart from being a curiosity of elasticity theory, may or may not have a deeper meaning. From a purely physical standpoint one is easily led to consider that, while the special theory of relativity is essentially bound to the finiteness of the velocity of information transfer, that is the light speed, in the mechanics of deformable solids the displacements in an isotropic elastic medium containing an incompressible dislocation in steady motion are limited by the acoustic wave speed. Therefore it does not seem surprising that in the equations of the motion of dislocations in continuous media the shear velocity takes the place of the velocity of light.

On the other hand, in the realm of continuum mechanics more complicated relativistic effects can be found. In fact, there are two velocities of information transfer even in isotropic solids, which are the velocities of shear and longitudinal waves. This simple fact implies that if we take into consideration moving edge dislocations instead of screw ones the equation of motion do not appear straight Lorentz-covariant ones [4,7]. This problem has been investigated by Weertman and Weertman [8] and Günter [9], who pointed out that the relativistic behaviour of the displacement vector field of a dislocation, solution of the elastic-dynamic Navier–Cauchy equations,

$$(\lambda + \mu)\,\mathrm{grad}(\mathrm{div}\,\mathbf{u}) + \mu \Delta_2 \mathbf{u} = \rho \frac{\partial^2 \mathbf{u}}{\partial t^2} \tag{1}$$

(where $\lambda$, $\mu$ and $\rho$ are the Lamé's moduli and the mass density of the medium, respectively) get complicated only if the dilatational part $\mathrm{div}\,\mathbf{u}$ of the displacement vector field does not vanish, differently from the case of a screw dislocation, whose statical solution is of pure shear type. Instead, by letting $\lambda$ go to infinity, which means that the medium is incompressible, a straightforward analogy with special relativity can be obtained. It must also be recalled that a theory aiming to describe the behaviour of continuous distributions of dislocations has been developed by Bilby and co-workers [10,11] with the support of non-Riemannian spaces and, eventually, polar media or Cosserat-like continua [12]. However, such a theory, more than contributing to the understanding of relativistic effects, can constitute a step in bridging the gap between micro and macro-mechanics. From a purely historical standpoint these findings can be linked to all the attempts of describing fundamental physics with continuum mechanics which were carried out in the 19th century with the scope of interpreting light propagation as transversal vibrations of an elastic medium [13]. These attempts ceased after the Michelson and Morley experiment which led scientists to drop the concept of aether. Nonetheless, this concept underlies many of the approaches to a more fundamental physical theory, mostly in the construction of a complete quantum field theory or in the quantum formulation of a gravity field theory. Among these proposals a leading example is that of Dirac's ''relativistic'' aether-based electrodynamics [14]. All these approaches to an aether-based quantum field theory employ classical or relativistic concepts derived by continuum mechanics, in which the topological structure of the underlying spacetime is not criticised nor further investigated. In spite of this, more sophisticated theoretical proposals for a fundamental theory of fields and particles have been recently formulated on the ground of transfinite manifolds, like *E*-infinity space [15,16]. In such theories the role of aether is played by new topological objects, as the ''cantorian dust'', which are based on fractal geometry.

Finally, several recent studies [17,18] which have taken into account nonlinearity, nonlocality and lattice discreteness, as well as involved atomistic simulations, have shown that in a three-dimensional medium dislocations can move faster than the speed of shear waves if they are created as supersonic dislocations at a strong stress concentration and are subjected to high shear stresses. In fact, as already anticipated by Eshelby [7] for edge dislocations, in the subsonic velocity range (namely below the shear wave speed, $c_T$) dislocation motion exhibits a fully relativistic regime while in the intersonic velocity range (namely between the shear wave speed, $c_T$, and the longitudinal wave speed, $c_L$) dislocation motion can occur above a critical value of the applied stress but is unstable below a certain critical speed [18]. Therefore a rigorous relativistic framework for the motion of soliton-like dislocations appears much more subtle and problematical than the special relativity one.

In order to give reason for attempting the construction of such a relativistic framework, it must be noticed that the dislocation theory, other than in impact, detonation, tectonic processes and many different branches of physics, plays an important role in metallic plasticity also. In fact the concept of dislocation has been acknowledged as the basis for a microscopic interpretation of plastic behaviour of metals since the pioneeristic work of Prandtl [19], who first conceived the convenience of describing plastic phenomena by means of a lattice model. In the second half of the past century several physicists made a substantial effort to set the originally scattered qualitative observations in dislocation mechanics into an ordered framework, making resort to various mathematical tools [10,11,20,21], but an explicit connection with the classical continuum theory of plasticity was not achieved notwithstanding the efforts produced about at the