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Bifurcations of forced oscillators with fuzzy uncertainties by the generalized cell mapping method

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Abstract

In this paper, bifurcations in dynamical systems with fuzzy uncertainties are studied by means of the fuzzy generalized cell mapping (FGCM) method. A bifurcation parameter is modeled as a fuzzy set with a triangular membership function. We first study a boundary crisis resulting from a collision of a fuzzy chaotic attractor with a fuzzy saddle on the basin boundary. The fuzzy chaotic attractor together with its basin of attraction is eradicated as the fuzzy control parameter reaches a critical point. We also show that a saddle-node bifurcation is caused by the collision of a fuzzy period-one attractor with a fuzzy saddle on the basin boundary. The fuzzy attractor together with its basin of attraction suddenly disappears as the fuzzy parameter passes through a critical value.

1. Introduction

There are uncertainties in engineering systems associated with the lack of precise knowledge of the system parameters and operating conditions and that are originated from the variabilities in manufacturing processes. The uncertainties can have significant influence on the dynamic response and the reliability of the system. This paper presents a method to analyze the response and bifurcation of nonlinear dynamical systems with fuzzy uncertainties.

Specifically, we are interested in a nonlinear dynamical system whose response is a fuzzy process, and study how the fuzzy response changes as the fuzzy parameter of the system varies. Bifurcation analysis of uncertain nonlinear dynamical systems is in general a difficult subject, partly because even the definition of bifurcation is open to discussion. Take the stochastic system as an example. The commonly accepted definition of the bifurcation is the ‘‘qualitative change’’of the system response as a control parameter varies. Meunier and Verga [1] studied pitchfork bifurcation of a stochastic dynamical system. They examined the quantities such as invariant measures, Lyapunov exponents, correlation functions, and exit times. It turns out that the behavior of all these quantities near the deterministic bifurcation point changes for different values of the bifurcation parameter, making them a poor indicator of bifurcation in some cases. They proposed an effective potential function of the invariant probability density function of the system response to describe the bifurcation and concluded that corresponding to the bifurcation point of the deterministic system, there is a bifurcation transition region for the stochastic system. Doi et al. [2] have also found that the invariant probability

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density of the system response is not indicative of bifurcation in some cases, and proposed to examine the qualitative changes of the spectrum of a Markov operator. The spectrum of the Markov operator reveals the bifurcation of the stochastic phase lockings of a van der Pol oscillator that does not show up in the invariant probability density of the system response. Other studies phenomenologically define stochastic bifurcation based on the observation of collision of stable attractors with saddle nodes [3–5].

For fuzzy nonlinear dynamical systems, the subject is even more difficult because the evolution of the membership function of the fuzzy response process is not readily obtained analytically. There is little study in the literature on the bifurcation of fuzzy nonlinear dynamical systems. There are studies of bifurcations of fuzzy control systems where the fuzzy control law leads to a nonlinear and deterministic dynamical system. The bifurcation studies are practically the same as that of deterministic systems [6,7]. The work in [8] deals with bifurcation of fuzzy dynamical systems having a fuzzy response. Numerical simulations are used to simulate the system response with a given parameter and fuzzy membership grade. The eigenvalues and the membership distribution are both used to describe the bifurcation. For a given membership grade, the bifurcation of the system is defined in the same manner as for the deterministic system.

The objective of this paper is to study bifurcations of fuzzy dynamical systems by the fuzzy generalized cell mapping (FGCM) method, which has been developed for nonlinear dynamical systems with fuzzy uncertainties [9]. The generalized cell mapping (GCM) method was first developed for global analysis of nonlinear dynamical systems by Hsu [10]. It should be noted that there have been many recent studies of the GCM method and applications. Song et al. [11] have developed new algorithms for applying the cell mapping method to higher dimensional systems and studied fuzzy control problems. Edwards and Choi [12] have proposed to impose the initial probability distribution in each cell that has the same form as a typical fuzzy membership function. The resulting generalized cell mapping, however, is still a Markov chain, while the generalized cell mapping for fuzzy systems in [9] is based on Zadeh's extension principle. A variety of random vibration and stochastic optimal problems have been studied by Sun and co-workers [13–15]. Crises and global bifurcations in deterministic [16–18] and stochastic [3–5] dynamical systems have been investigated using the GCM with digraphs. The authors have recently proposed a fuzzy generalized cell mapping (FGCM) method for the analysis of a class of bifurcations of fuzzy nonlinear dynamical systems involving merging of two stable or unstable solutions into one, and considered only one-dimensional systems [19]. The current paper is an extension of our early work and will study fuzzy bifurcations of two interesting nonlinear dynamical systems in higher dimensional phase space. In particular, we shall study boundary crisis and saddle-node bifurcations involving collision of a stable attractor with a saddle.

The remainder of the paper is outlined as follows. In Section 2, we describe the FGCM method, and discuss its properties. In Section 3, we study fuzzy bifurcations of two nonlinear dynamical systems. The paper concludes in Section 4.

2. The method

2.1. Fuzzy generalized cell mapping

Here, we first review the FGCM method for nonlinear dynamical systems with fuzzy uncertainties [9]. Consider a dynamical system with a fuzzy parameter

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t, S), \quad \mathbf{x} \in \mathbf{D}, \tag{1}$$

where $\mathbf{x}$ is the state vector, t the time variable, S a fuzzy set with a membership function $\mu_S(s) \in (0,1]$ where $s \in S$, and $\mathbf{f}$ is a vector-valued nonlinear function of its arguments. It is assumed to be periodic in t with period T for all $s \in S$ and to satisfy the Lipschitz condition for all $s \in S$, and $\mathbf{D}$ is a bounded domain of interest in the state space. A fuzzy Poincaré map can be obtained from Eq. (1) as

$$\mathbf{x}(n+1) = \mathbf{G}(\mathbf{x}(n), S), \quad n = 0, 1, 2, \ldots \tag{2}$$

The cell mapping method proposes to further discretize the state space in searching for the global solution of the system [10]. In order to apply the cell mapping method, we also need to discretize the fuzzy set S. Suppose that S is a finite interval in R. We divide S into M segments of appropriate length and sample a value $s_k \in S$ $(k = 1, \ldots, M)$ in the middle of each segment. The division of S is such that there is at least one s_k with membership grade equal to one. The domain $\mathbf{D}$ is then discretized into N small cells. Each cell is identified by an integer ranging from 1 to N. For a cell, say cell j, N_p points are uniformly sampled from cell j, $M \times N_p$ fuzzy sample trajectories are computed for one period T, or one mapping step. Each trajectory carries a membership grade determined by that of s_k's. We then find the cells in which the end points of the trajectories fall. Assume that cell i is one of the image cells of cell j, and that there are m $(0 < m \leqslant MN_p)$ trajectories falling in cell i. Define a quantity