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Crisis of interspike intervals in Hodgkin–Huxley model

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Abstract

The bifurcations of the chaotic attractor in a Hodgkin–Huxley (H–H) model under stimulation of periodic signal is presented in this work, where the frequency of signal is taken as the controlling parameter. The chaotic behavior is realized over a wide range of frequency and is visualized by using interspike intervals (ISIs). Many kinds of abrupt undergoing changes of the ISIs are observed in different frequency regions, such as boundary crisis, interior crisis and merging crisis displaying alternately along with the changes of external signal frequency. And there are logistic-like bifurcation behaviors, e.g., periodic windows and fractal structures in ISIs dynamics. The saddle-node bifurcations resulting in collapses of chaos to period-6 orbit in dynamics of ISIs are identified.

1. Introduction

There has been a continuing argument on neuronal spike trains which play an important role in encoding and decoding the neuronal information. As we all know, the neural systems have strong nonlinear characters and usually able to display different dynamics according to system parameters or external inputs in ISIs sequence. When these parameters are slightly modified, the system's dynamics usually experience also little modification, except when these changes occur in the vicinity of a critical point. In that case an abrupt qualitative change or transition in the dynamics occurs. These transitions, for example, may be from periodic to chaotic, from chaotic to chaotic, or in their inverse [1].

The role of chaotic activities in the brain is growingly interested in neuroscience and chaotic theory. For instance, the numerical evidence and theoretical reasoning has proved that there is a chaos–chaos transition in the brain, in which the change of the attractor size is sudden but continuous, different from general discontinues chaos–chaos transitions, and which occurs in the Hindmarsh–Rose model of a neuron. This transition corresponds to different neural dynamics, i.e. the chaotic dynamics of bursting or spiking dynamics [2]. The crisis of the thermally sensitive neuron resulted from homoclinic bifurcation of a saddle-focus fixed point which is embedded in the chaotic attractors, also been studied in Ref. [3]. Similarly, the external shifted stimulus current will induce the chaos collapsing to a period-3 orbit in the dynamics of a quadratic logistic map neuron [4]. Lee and Farhat [5,6] suggested that the bifurcating neuron's bistability and associative memory is related to attractor-merging crisis; and Xie et al. [7] introduced periodic orbit theory to

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characterize the dynamical behavior of aperiodic firing neurons, and considered that bifurcations, crises and sensitive dependence of chaotic motions on control parameters can be the underlying mechanisms.

There are many chaotic activities that have been observed in experimental studies of electroencephalogram (EEG) signals and ISIs sequence. For example, Xie et al. [8] have studied variations of chaos during an epileptic seizure; Freeman and Skarda [9] observed the olfactory system of the rabbit that the nervous activity of the olfactory system switches from a chaotic to a periodic state whenever a familiar odor is detected. This experimental observation stimulated their reflection on the role of chaos in perception processes and led them to postulate that chaos can serve as the ground state of a perception process, i.e. an elevated state that has quick transition routes to many periodic states [5]; and onset of the explosion occurs of ISIs and at least two periodic windows have been observed in cases of increasing and decreasing temperature of the saline bath for crayfish caudal photoreceptor cell [3].

The study of transitions between different dynamic behaviors in nonlinear systems is an issue of major interest for the theory of nonlinear dynamics and chaos [1,10,11]. That has also been widely interested in by biophysicist. The observation of bifurcations and crisis in this work is relevant both to the theory of nonlinear dynamics and chaos, and to biophysics, also, particularly to neurobiology. Chaos–chaos transitions will help us to understand how the neural system is able to give quick responses to the different external or internal stimulus, i.e., rapid switches between different neuronal dynamic activities.

2. The Hodgkin–Huxley (H–H) model

As is well known, the H–H model equations have been derived from a squid giant axon. These equations can describe the spiking behavior and refractoriness of real neuron very well [12,13], so that this kind of model is employed in this work. The H–H model for the action potential of a space-clamped squid axon is defined by the four-dimensional vector field [14]

$$\begin{cases} \dot{u} = I_{\text{ext}} - [120m^3h(u+115) + 36n^4(u-12) + 0.3(u+10.6)], \\ \dot{m} = (1-m)\Psi\left(\frac{u+25}{10}\right) - m\left(4\exp\left(\frac{u}{18}\right)\right), \\ \dot{n} = 0.1(1-n)\Psi\left(\frac{u+10}{10}\right) - n\left(0.125\exp\left(\frac{u}{80}\right)\right), \\ \dot{h} = 0.07(1-h)\Psi\left(\frac{u}{20}\right) - h\left(\frac{1}{1+\exp\left(\frac{u+30}{10}\right)}\right), \end{cases} \tag{1}$$

where

$$\Psi(x) = \frac{x}{\exp(x) - 1} \tag{2}$$

and variables u, m, n, and h represent membrane potential, activation of a sodium current, activation of a potassium current, and inactivation of the sodium current. There is also a current parameter I_{ext} that is an external periodic signal current into the space-clamped axon in this work, i.e.

$$I_{\text{ext}} = I_{\text{shift}} + \sin(2\pi f_0 t), \tag{3}$$

where $I_{\text{shift}} = 10\ \mu\text{A/cm}^2$, being the amplitude of current shift, and $f_0 = 1/3$ Hz being the basic stimulus frequency in this work.

Recalling that the H–H convention for membrane potential reverses the sign from modern conventions, the voltage spikes of action potentials should be negative in the H–H model. When improved models for the membrane potential of the squid axon have been formulated, the H–H model remains the paradigm for conductance-based models of neural systems. From a mathematical viewpoint, varied properties of the dynamics of the H–H vector field have been studied. Nonetheless, we remain far from a comprehensive understanding of the dynamics displayed by this vector field.

In this work, the ordinary differential equations (1) are integrated by using double precision fourth-order Runge–Kutta method, with integration time step 0.01, the rest membrane potential equals to 0 mV.

3. Bifurcations and crisis of ISIs in the H–H model

The H–H model has been simulated numerically in the absence of noise, using the ISIs as a state variable. The ISIs are registered by the membrane potential crossing a threshold (at 60 mV) with positive derivative (Poincaré surface of