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A generalized F-expansion method to find abundant families of Jacobi Elliptic Function solutions of the (2 + 1)-dimensional Nizhnik–Novikov–Veselov equation

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Abstract

In the present paper, a generalized F-expansion method is proposed by further studying the famous extended F-expansion method and using a generalized transformation to seek more types of solutions of nonlinear partial differential equations. With the aid of symbolic computation, we choose (2 + 1)-dimensional Nizhnik–Novikov–Veselov equations to illustrate the validity and advantages of the method. As a result, abundant new exact solutions are obtained including Jacobi Elliptic Function solutions, soliton-like solutions, trigonometric function solution etc. The method can be also applied to other nonlinear partial differential equations.

1. Introduction

In recent years, nonlinear partial differential equations (NPDEs) have attracted considerable attention. A vast variety of the powerful and direct methods to find all kinds of analytic solutions of NPDEs have been developed. As is known to all, there are many kinds of powerful methods to obtain the solutions of NPDEs. For example, in the past decades, there has been significant progression in the development of these methods such as extended F-expansion method [1], sinh–cosh method [2], extended Jacobi Elliptic Function expansion method [3], inverse scattering method [4], Backlund transformation [5–8], Darboux transformation [9–11], Hirota bilinear method [11–13], algebro-geometric method [14,15] and tanh-function method [16], the general tanh-function method [17] etc.

In this paper, we present a generalized F-expansion method for finding more types exact solutions of NPDEs. By using this method via symbolic computation system MAPLE, we obtain new and more general solutions of the (2 + 1)-dimensional Nizhnik–Novikov–Veselov equations in [18]. The solutions we have got are more abundant than the solutions in [2]. In other words, the solutions we get contain the solutions in [2] and ours are more general than the ones from the extended F-expansion method. At the same time, our method is more convenient than the method

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in [3]. In fact, our method is also powerful to solve other NPDEs and can help to get many new exact solutions which we have never seen before within our knowledge.

The rest of the paper is organized as follows. In Section 2, a new generalized F-expansion method is presented. Then, in Section 3, we choose (2 + 1)-dimensional Nizhnik–Novikov–Veselov equations to illustrate the validity and advantages of the method. As a result, many types of new solutions are obtained. Finally, some conclusions and discussions are given in Section 4.

2. The generalized F-expansion method

In this section, we will give the detailed description of our method (called generalized F-expansion method). Considering a given NPDEs with independent variables $X = (t, x_1, x_2, \ldots, x_m)$ and dependent variable u, v, w:

$$\begin{cases} F(u, v, w, u_t, v_t, w_t, u_{x_1}, v_{x_1}, w_{x_1}, u_{tt}, v_{tt}, w_{tt}, \ldots) = 0, \\ G(u, v, w, u_t, v_t, w_t, u_{x_1}, v_{x_1}, w_{x_1}, u_{tt}, v_{tt}, w_{tt}, \ldots) = 0, \\ H(u, v, w, u_t, v_t, w_t, u_{x_1}, v_{x_1}, w_{x_1}, u_{tt}, v_{tt}, w_{tt}, \ldots) = 0. \end{cases} \tag{1}$$

The solutions of Eqs. (1) will be sought by new and more general ansätz:

$$\begin{cases} u(X) = a_0 + \sum\limits_{i=1}^{n_1} (f(\omega))^{i-1} \left(a_{i1} f(\omega) + a_{i2} g(\omega) + \frac{a_{i3}}{f(\omega)} \right), \quad a_{i1}^2 + a_{i2}^2 + a_{i3}^2 \neq 0, \\ v(X) = b_0 + \sum\limits_{j=1}^{n_2} (f(\omega))^{j-1} \left(b_{j1} f(\omega) + b_{j2} g(\omega) + \frac{b_{j3}}{f(\omega)} \right), \quad b_{j1}^2 + b_{j2}^2 + b_{j3}^2 \neq 0, \\ w(X) = c_0 + \sum\limits_{k=1}^{n_3} (f(\omega))^{k-1} \left(c_{k1} f(\omega) + c_{k2} g(\omega) + \frac{c_{k3}}{f(\omega)} \right), \quad c_{k1}^2 + c_{k2}^2 + c_{k3}^2 \neq 0, \end{cases} \tag{2}$$

where n_i (i = 1, 2, 3) is an integers respectively which is determined by balancing the highest order derivative terms with the nonlinear terms in the given Eqs. (1). When $\omega = \omega(X)$, the function $f(\omega)$ and $g(\omega)$ satisfy the following relations:

$$\begin{cases} f'^2(\omega) = l_1 f^4(\omega) + m_1 f^2(\omega) + n_1, \\ g'^2(\omega) = l_2 g^4(\omega) + m_2 g^2(\omega) + n_2, \\ g^2(\omega) = \dfrac{l_1 f^2(\omega)}{l_2} + \dfrac{m_1 - m_2}{3 l_2}, \\ n_1 = \dfrac{m_1^2 - m_2^2 + 3 l_2 n_2}{3 l_1}, \end{cases} \tag{3}$$

where $f' = \frac{d}{d\omega} f(\omega)$, $g' = \frac{d}{d\omega} g(\omega)$, and ω, a_0, a_{i1}, a_{i2}, a_{i3}, b_0, b_{j1}, b_{j2}, b_{j3}, c_0, c_{k1}, c_{k2}, c_{k3}, ($i = 1, 2, \ldots, n_1$, $j = 1, 2, \ldots, n_2$, $k = 1, 2, \ldots, n_3$) are all differentiable functions of $X = (t, x_1, x_2, \ldots, x_m)$ to be determined later.

Over here, it is necessary for us to point out that the ansätz:

$$u(X) = \sum_{j=0}^{n} \sum_{i=0}^{j} c_{ji} (F(\omega))^i (G(\omega))^{j-i} \tag{4}$$

in [1] is not proper because $F(\omega)$ and $G(\omega)$ satisfy

$$G^2(\omega) = \frac{l_1 F^2(\omega)}{l_2} + \frac{m_1 - m_2}{3 l_2} \tag{5}$$

namely $G^2(\omega)$ can be expressed by $F^2(\omega)$. For example, when $n = 2$ the ansätz (4) becomes

$$u(X) = c_{00} + c_{10} G(\omega) + c_{11} F(\omega) + c_{20} G^2(\omega) + c_{21} F(\omega) G(\omega) + c_{22} F^2(\omega) \tag{6}$$

over here $G^2(\omega)$ is redundant and if there is $G^2(\omega)$, it is likely to get the ordinary solutions to Eqs. (1). So in our paper we use the transform (2).

In addition, we should point out that in [1] using the ansätz (4) may be get the useless and unwanted solutions to Eqs. (1).

Substituting (2) into the given Eqs. (1) with (3) and collecting coefficients of polynomials of $f(\omega)$, $g(\omega)$, $f'(\omega)$ (where $f' = \frac{d}{d\omega} f(\omega)$), with the aid of Maple, then setting each coefficient to zero, we can deduce a set of over-determined partial differential equations. With the help of Maple, solve the equations, then we can determine ω, a_0, a_{i1}, a_{i2}, a_{i3}, b_0, b_{j1}, b_{j2}, b_{j3}, c_0, c_{k1}, c_{k2}, c_{k3}, ($i = 1, 2, \ldots, n_3$, $j = 1, 2, \ldots, n_3$, $k = 1, 2, \ldots, n_3$), at the same time substitute different kinds of Jacobi Elliptic Functions into (3) to get the relations between the parameters and the modulus K of Jacobi Elliptic Function.