Journal of Geometry and Physics 106 (2016) 184–191

Contents lists available at ScienceDirect

Journal of Geometry and Physics

journal homepage: www.elsevier.com/locate/jgp

Orbifold E-functions of dual invertible polynomials

Wolfgang Ebeling [a,*], Sabir M. Gusein-Zade [b], Atsushi Takahashi [c]

[a] Institut für Algebraische Geometrie, Leibniz Universität Hannover, Postfach 6009, D-30060 Hannover, Germany
[b] Moscow State University, Faculty of Mechanics and Mathematics, Moscow, GSP-1, 119991, Russia
[c] Department of Mathematics, Graduate School of Science, Osaka University, Toyonaka Osaka, 560-0043, Japan

ARTICLE INFO

Article history:
Received 23 September 2015
Received in revised form 25 February 2016
Accepted 22 March 2016
Available online 1 April 2016

MSC:
32S25
32S35
14J33
14L30

Keywords:
Mirror symmetry
Singularity
Mixed Hodge structure
Monodromy
Orbifold
E-function

ABSTRACT

An invertible polynomial is a weighted homogeneous polynomial with the number of monomials coinciding with the number of variables and such that the weights of the variables and the quasi-degree are well defined. In the framework of the search for mirror symmetric orbifold Landau–Ginzburg models, P. Berglund and M. Henningson considered a pair (f, G) consisting of an invertible polynomial f and an abelian group G of its symmetries together with a dual pair $(\widetilde{f}, \widetilde{G})$. We consider the so-called orbifold E-function of such a pair (f, G) which is a generating function for the exponents of the monodromy action on an orbifold version of the mixed Hodge structure on the Milnor fibre of f. We prove that the orbifold E-functions of Berglund–Henningson dual pairs coincide up to a sign depending on the number of variables and a simple change of variables. The proof is based on a relation between monomials (say, elements of a monomial basis of the Milnor algebra of an invertible polynomial) and elements of the whole symmetry group of the dual polynomial.

© 2016 Elsevier B.V. All rights reserved.

0. Introduction

Mirror symmetry is the famous observation by physicists that there exist pairs of Calabi–Yau manifolds whose Hodge diamonds are symmetric in a certain sense. P. Berglund and T. Hübsch [1] suggested a method to construct such mirror symmetric pairs of Calabi–Yau manifolds. They considered a polynomial f of a special form, a so called *invertible* one, and its *Berglund–Hübsch transpose* $\widetilde{f}$: see below. The manifolds suggested in [1] were resolutions of the following ones: the hypersurface in a weighted projective space defined by the equation $f = 0$ and the quotient of the hypersurface in a weighted projective space defined by the equation $\widetilde{f} = 0$ by a certain group of symmetries of the polynomial $\widetilde{f}$. In [1] these polynomials appeared as potentials of Landau–Ginzburg models. In [2] this construction was generalized to an orbifold setting. An orbifold Landau–Ginzburg model of [2] was described by a pair (f, G), where f is an invertible polynomial and G is a (finite) abelian group of symmetries of f. For a pair (f, G) they defined the dual pair $(\widetilde{f}, \widetilde{G})$. There were observed some symmetries of the manifolds constructed in this way. Namely, it was observed that the elliptic genera of dual pairs coincide up to sign [2,3]. In [2], there was defined the Poincaré polynomial of a pair (f, G) as a particular limit of the elliptic genus of the corresponding orbifold. It is a fractional power polynomial in one variable. The paper [2] contains a physical style proof of the fact that the Poincaré polynomials of dual pairs coincide up to sign.

* Corresponding author.
E-mail addresses: ebeling@math.uni-hannover.de (W. Ebeling), sabir@mccme.ru (S.M. Gusein-Zade), takahashi@math.sci.osaka-u.ac.jp (A. Takahashi).

http://dx.doi.org/10.1016/j.geomphys.2016.03.026
0393-0440/© 2016 Elsevier B.V. All rights reserved.

Certain aspects of Landau–Ginzburg models are related to singularity theory. In [4] (see also [5]), for a pair (f, G) with the group G satisfying some restrictions, there was defined a so called orbifold E-function of the pair which is a generating function for the exponents of the monodromy action on an orbifold version of the mixed Hodge structure on the Milnor fibre of f. It turns out that it agrees formally with a two-variables generalization of the Poincaré polynomial. Here we extend this definition to an arbitrary group G. There exists a conjectural symmetry property of it for Berglund–Henningson dual pairs (see Theorem 9). It seems that this symmetry property was understandable for specialists in Landau–Ginzburg models, however to our knowledge there did not exist a proof of it. Here we give a simple and readable proof of this symmetry property and therefore also a new proof of the statement of P. Berglund and M. Henningson. The method of the proof is different from that for the Poincaré polynomial in [2]. It is based on a relation between monomials (say, elements of a monomial basis of the Milnor algebra of an invertible polynomial) and elements of the maximal symmetry group of the dual polynomial. This relation was essentially described in [6]. It was generalized by M. Krawitz [7] to monomials invariant under a so called admissible subgroup G of the maximal symmetry group G_f of f and its dual group $\widetilde{G}$ (see also [4, Proposition 3.6]). Our proof of the symmetry property applies to any subgroup G of the maximal symmetry group G_f of f, not only to admissible ones. Moreover, as an intermediate step we derive a formula for the orbifold E-function expressing it in a symmetric way in terms of the group and the dual group (see Proposition 14).

The described symmetry of the E-functions implies, in particular, the statement obtained earlier that the reduced orbifold zeta functions of dual pairs either coincide or are inverse to each other (depending on the number of variables): [8].

1. Invertible polynomials

Let $f(x_1, \dots, x_n)$ be a non-degenerate weighted homogeneous polynomial, namely, a polynomial with an isolated singularity at the origin with the property that there are positive integers $w_1, \dots, w_n$ and d such that $f(\lambda^{w_1}x_1, \dots, \lambda^{w_n}x_n) = \lambda^d f(x_1, \dots, x_n)$ for $\lambda \in \mathbb{C}^*$. We call $(w_1, \dots, w_n; d)$ a system of *weights*.

Definition. A non-degenerate weighted homogeneous polynomial $f(x_1, \dots, x_n)$ is called *invertible* if the following conditions are satisfied:

(i) the number of variables $(=n)$ coincides with the number of monomials in the polynomial $f(x_1, \dots, x_n)$, namely,

$$f(x_1, \dots, x_n) = \sum_{i=1}^{n} c_i \prod_{j=1}^{n} x_j^{E_{ij}}$$

for some coefficients $c_i \in \mathbb{C}^*$ and non-negative integers E_{ij} for $i, j = 1, \dots, n$,

(ii) a system of weights $(w_1, \dots, w_n; d)$ can be uniquely determined by the polynomial $f(x_1, \dots, x_n)$ up to a constant factor $\gcd(w_1, \dots, w_n; d)$, namely, the matrix $E := (E_{ij})$ is invertible over $\mathbb{Q}$.

Without loss of generality one may assume that $c_i = 1$ for $i = 1, \dots, n$. This can be achieved by rescaling the variables.

According to [9], an invertible polynomial f is a (Thom–Sebastiani) sum of invertible polynomials (in groups of different variables) of the following types:

(1) $x_1^{a_1}x_2 + x_2^{a_2}x_3 + \cdots + x_{m-1}^{a_{m-1}}x_m + x_m^{a_m}$ (chain type; $m \geq 1$);
(2) $x_1^{a_1}x_2 + x_2^{a_2}x_3 + \cdots + x_{m-1}^{a_{m-1}}x_m + x_m^{a_m}x_1$ (loop type; $m \geq 2$).

Remark 1. In [9] the authors distinguished also polynomials of the so called Fermat type: $x_1^{a_1}$. One can regard the Fermat type polynomial $x_1^{a_1}$ as a chain type polynomial with $m = 1$.

Let $f(x_1, \dots, x_n) = \sum_{i=1}^{n} c_i \prod_{j=1}^{n} x_j^{E_{ij}}$ be an invertible polynomial. Define rational numbers $q_1, \dots, q_n$ by the unique solution of the equation

$$E \begin{pmatrix} q_1 \\ \vdots \\ q_n \end{pmatrix} = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}.$$

One can easily see that $q_i = w_i/d$, $i = 1, \dots, n$, for the system of weights $(w_1, \dots, w_n; d)$.

Definition. Let $f(x_1, \dots, x_n) = \sum_{i=1}^{n} c_i \prod_{j=1}^{n} x_j^{E_{ij}}$ be an invertible polynomial. The *group of diagonal symmetries* G_f of f is the finite abelian group defined by

$$G_f = \left\{(\lambda_1, \dots, \lambda_n) \in (\mathbb{C}^*)^n \mid f(\lambda_1 x_1, \dots, \lambda_n x_n) = f(x_1, \dots, x_n)\right\}.$$

In other words

$$G_f = \left\{(\lambda_1, \dots, \lambda_n) \in (\mathbb{C}^*)^n \;\middle|\; \prod_{j=1}^{n} \lambda_j^{E_{1j}} = \cdots = \prod_{j=1}^{n} \lambda_j^{E_{nj}} = 1\right\}.$$

Download English Version:

https://daneshyari.com/en/article/1894551

Download Persian Version:

https://daneshyari.com/article/1894551

Daneshyari.com