Journal of Geometry and Physics 86 (2014) 101–111

Contents lists available at ScienceDirect

Journal of Geometry and Physics

journal homepage: www.elsevier.com/locate/jgp

Black brane solutions governed by fluxbrane polynomials

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ARTICLE INFO

Article history:
Received 2 January 2014
Accepted 10 July 2014
Available online 21 July 2014

Keywords:
Black branes
Black holes
Toda chains
Lie algebras
Polynomials

ABSTRACT

A family of composite black brane solutions in the model with scalar fields and fields of forms is presented. The metric of any solution is defined on a manifold which contains a product of several Ricci-flat "internal" spaces. The solutions are governed by moduli functions H_s ($s = 1, \dots, m$) obeying non-linear differential equations with certain boundary conditions imposed. These master equations are equivalent to Toda-like equations and depend upon the non-degenerate $(m \times m)$ matrix A. It was conjectured earlier that the functions H_s should be polynomials if A is a Cartan matrix for some semisimple finite-dimensional Lie algebra (of rank m). It is shown that the solutions to master equations may be found by using so-called fluxbrane polynomials which can be calculated (in principle) for any semisimple finite-dimensional Lie algebra. Examples of dilatonic charged black hole (0-brane) solutions related to Lie algebras A_1, A_2, C_2 and G_2 are considered.

1. Introduction

In this paper we deal with spherically-symmetric solutions with horizon defined on product manifolds containing several Ricci-flat factor-spaces (with diverse signatures and dimensions). Solutions of such type appear either in models with antisymmetric forms and scalar fields [1–11] or in models with multi-component anisotropic fluid (MCAF) [12–16]. For black brane solutions with 1-dimensional factor-spaces (of Euclidean signatures) see [17–19] and references therein.

These and more general brane cosmological and spherically symmetric solutions were obtained by reduction of the field equations to the Lagrange equations corresponding to Toda-like systems [2,20].

Here we consider black brane solutions in the model with scalar field and fields of forms, when certain relations on parameters are imposed. The solutions are governed by a set of functions H_s, $s = 1, \dots, m$, obeying non-linear differential equations with certain boundary conditions imposed. These equations depend upon the non-degenerate $m \times m$ matrix A. It was conjectured in [6] that the moduli functions H_s should be polynomials when A is a Cartan matrix for some semisimple finite-dimensional Lie algebra $\mathcal{G}$ of rank m. In this case we deal with special solutions to open Toda chain equations related to the Lie algebra $\mathcal{G}$ [21–24] which are integrable in quadratures. The conjecture from [6] was verified for the Lie algebras A_m, C_{m+1}, $m \geq 1$ in [7,8] by using the solutions to Toda chain equations corresponding to the Lie algebras A_m from [25].

Here we show that the black brane solutions under consideration may be also found by using so-called fluxbrane polynomials [26], which may be calculated (in principle) for any (semi)simple finite-dimensional Lie algebra using MATHEMATICA [27] or MAPLE [28,29]. For any solution we find the Hawking temperature as a function of p_i-parameters of fluxbrane polynomials. Recently, in [30] a similar approach appeared in a context of special Toda charged black hole solutions corresponding to Lie algebras A_m, where a formal relation for A_m fluxbrane polynomials was obtained in a way

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http://dx.doi.org/10.1016/j.geomphys.2014.07.015

similar to our earlier consideration [7,8] based on the Anderson solution [25]. Here we illustrate the general approach by applying it to dilatonic charged black hole solutions related to Lie algebras A_1, A_2, C_2 and G_2.

2. Black brane solutions

We start with a model governed by the action

$$S = \int d^Dx\sqrt{|g|}\Big\{R[g] - h_{\alpha\beta}g^{MN}\partial_M\varphi^\alpha\partial_N\varphi^\beta - \sum_{a\in\triangle}\frac{\theta_a}{n_a!}\exp[2\lambda_a(\varphi)](F^a)^2\Big\}, \tag{2.1}$$

where $g = g_{MN}(x)dx^M \otimes dx^N$ is a metric, $\varphi = (\varphi^\alpha) \in \mathbb{R}^l$ is a vector of scalar fields, $(h_{\alpha\beta})$ is a constant symmetric non-degenerate $l \times l$ matrix ($l \in \mathbb{N}$), $\theta_a = \pm 1$,

$$F^a = dA^a = \frac{1}{n_a!}F^a_{M_1\dots M_{n_a}}dz^{M_1}\wedge\cdots\wedge dz^{M_{n_a}} \tag{2.2}$$

is a n_a-form ($n_a \geq 1$), λ_a is a 1-form on $\mathbb{R}^l$: $\lambda_a(\varphi) = \lambda_{a\alpha}\varphi^\alpha$, $a \in \triangle$, $\alpha = 1,\dots,l$. In (2.1) we denote $|g| = |\det(g_{MN})|$, $(F^a)^2_g = F^a_{M_1\dots M_{n_a}}F^a_{N_1\dots N_{n_a}}g^{M_1N_1}\dots g^{M_{n_a}N_{n_a}}$, $a \in \triangle$. Here $\triangle$ is some finite set. In the models with one time all $\theta_a = 1$ when the signature of the metric is $(-1,+1,\dots,+1)$.

In [6–8] we have obtained a family of black brane solutions to the field equations corresponding to the action (2.1). These solutions are defined on the manifold

$$M = (R_0, +\infty)\times(M_0 = S^{d_0})\times(M_1 = \mathbb{R})\times\cdots\times M_n, \tag{2.3}$$

and have the following form

$$g = \Big(\prod_{s\in S}H_s^{2h_sd(I_s)/(D-2)}\Big)\Big\{f^{-1}dR\otimes dR + R^2g^0 - \Big(\prod_{s\in S}H_s^{-2h_s}\Big)fdt\otimes dt + \sum_{i=2}^{n}\Big(\prod_{s\in S}H_s^{-2h_s\delta_{iI_s}}\Big)g^i\Big\}, \tag{2.4}$$

$$\exp(\varphi^\alpha) = \prod_{s\in S}H_s^{h_s\chi_s\lambda^\alpha_{a_s}}, \tag{2.5}$$

$$F^a = \sum_{s\in S}\delta^a_{a_s}\mathcal{F}^s, \tag{2.6}$$

where $f = 1 - 2\mu/R^d$,

$$\mathcal{F}^s = -\frac{Q_s}{R^{d_0}}\left(\prod_{s'\in S}H_{s'}^{-A_{ss'}}\right)dR\wedge\tau(I_s), \quad s\in S_e, \tag{2.7}$$

$$\mathcal{F}^s = Q_s\tau(\bar{I}_s), \quad s \in S_m. \tag{2.8}$$

Here $Q_s \neq 0$, $s \in S$, are charge densities, $R > R_0$, $R_0 = (2\mu)^{1/d} > 0$ and $d = d_0 - 1$.

In (2.4) g^0 is the canonical metric on the unit sphere $M_0 = S^{d_0}$ and g^i is a Ricci-flat metric on M_i, $i = 2,\dots,n$ and $\delta_{iI} = \sum_{j\in I}\delta_{ij}$ is the indicator of i belonging to I: $\delta_{iI} = 1$ for $i \in I$ and $\delta_{iI} = 0$ otherwise. We also denote $g^1 = -dt\otimes dt$.

The brane set S is by definition

$$S = S_e \cup S_m, \quad S_v = \cup_{a\in\triangle}\{a\}\times\{v\}\times\Omega_{a,v}, \tag{2.9}$$

$v = e, m$ and $\Omega_{a,e}, \Omega_{a,m} \subset \Omega$, where $\Omega = \Omega(n)$ is the set of all non-empty subsets of $\{1,\dots,n\}$, i.e. all branes do not "live" in M_0.

Any brane index $s \in S$ has the form $s = (a_s, v_s, I_s)$, where $a_s \in \triangle$, $v_s = e, m$ and $I_s \in \Omega_{a_s,v_s}$. The sets S_e and S_m define electric and magnetic branes correspondingly. In (2.5) $\chi_s = +1, -1$ for $s \in S_e, S_m$, respectively. All branes contain the time manifold $M_1 = \mathbb{R}$, i.e.

$$1 \in I_s, \quad \forall s \in S. \tag{2.10}$$

All manifolds M_i, $i > 0$, are assumed to be oriented and connected and the volume d_i-forms

$$\tau_i \equiv \sqrt{|g^i(y_i)|}\,dy_i^1\wedge\cdots\wedge dy_i^{d_i}, \tag{2.11}$$

and signature parameters

$$\varepsilon(i) \equiv \mathrm{sign}(\det(g^i_{m_in_i})) = \pm 1 \tag{2.12}$$